

T A B L E S

AND

T R A C T S.

T. A. B. L. S.

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T. R. A. C. T. S.

T A B L E S

AND

T R A C T S,

RELATIVE TO

Several ARTS and SCIENCES.

By JAMES FERGUSON, F. R. S.

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With ADDITIONS.

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ADVERTISEMENT.

THE usefulness of numerical Tables, in the practical arts and sciences, is so universally acknowledged, that the present publication scarce needs an apology. By this means we save a vast deal of time and labour: witness the facility with which the operations of Trigonometry, and the more difficult questions of arithmetic, are now performed, when compared with the ope-rose and discouraging methods of computation used in ancient times.

No wonder therefore that the Trigonometrical Tables, and those of Logarithms, reduced to the most perfect form, by the successive labours of learned and ingenious men, are in every body's hands. But still there are many particular Tables and Tracts, relative to useful Arts and Sciences, which lie scattered in different volumes, some in print and some in manuscript, to which many curious persons cannot always have
ready

ready access. Such of these as the author judged would be most acceptable to the public, he hath collected into this manual, together with a few easy rules and examples directing their use. To these he hath added several of his own: and, throughout the Tables, he hath taken all possible care that the numbers should be correct.

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TABLE I.

The mean time of New Moon in January, according to the Old Stile.

Julian Years.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	D.	H.	M.	S.	°	'	S.	°	'	S.	°	'
1700	8	14	45	6	21	14	0	0	55	4	13	9
01	27	12	17	7	10	7	11	6	32	5	21	52
02	16	21	6	6	29	22	9	16	20	5	29	55
03	6	5	55	6	18	38	7	25	8	6	7	57
04	24	3	27	7	7	1	7	1	45	7	16	41
05	13	12	16	6	26	16	5	11	33	8	24	43
06	2	21	4	6	15	32	3	21	21	8	2	46
07	21	18	37	7	3	35	2	26	58	9	11	29
08	10	3	25	6	23	11	1	6	46	9	19	32
09	29	0	58	7	11	33	0	12	23	10	28	15
1710	18	19	47	7	0	48	10	22	12	11	6	18
11	7	18	35	6	20	3	9	2	0	11	14	20
12	25	16	8	7	8	26	8	7	37	0	23	3
13	15	0	56	6	27	42	6	17	25	1	1	6
14	4	9	45	6	16	57	4	27	13	1	9	9
15	23	7	17	7	5	20	4	2	50	2	17	52
16	11	16	6	6	24	35	2	12	38	2	25	54
17	1	0	54	6	13	51	0	22	26	3	3	37
18	19	22	27	7	2	13	11	28	3	4	12	40
19	9	7	16	6	21	29	10	7	51	4	20	43
1720	27	4	48	7	9	51	9	13	28	5	29	26
21	16	13	37	6	29	7	7	23	16	6	7	29
22	5	22	25	6	18	23	6	3	4	6	15	31
23	24	19	58	7	6	45	5	8	41	7	24	14
24	13	4	46	6	26	1	3	18	29	8	2	17
25	2	13	35	6	15	17	1	28	17	8	10	20
26	21	11	8	7	3	39	1	3	54	9	19	3
27	10	19	56	6	22	55	11	13	42	9	27	6
28	28	17	29	7	11	17	10	19	19	11	5	49
29	18	2	17	7	0	33	8	29	7	11	13	51
1730	7	11	6	6	19	49	7	8	55	11	21	54
31	26	8	38	7	8	11	6	14	33	1	0	37
32	14	17	27	6	27	27	4	24	29	1	8	40

TABLE I. *continued.*
Old Stile.

Julian Years.	New Moon.			Sun's Anomaly.	Moon's Anomaly.	Sun from Node.
	D.	H.	M.	S. ° '	S. ° '	S. ° '
1733	4	2	16	6 16 43	3 4 9	1 16 43
34	22	23	48	7 5 5	2 9 46	2 25 26
35	12	8	37	6 14 21	0 19 34	3 3 28
36	0	17	25	6 13 36	10 29 22	3 11 31
37	19	14	58	7 1 58	10 4 59	4 20 14
38	8	23	47	6 21 14	8 14 47	4 28 17
39	27	21	19	7 9 36	7 20 24	6 7 0
1740	16	6	8	6 28 52	6 0 12	6 15 3
41	5	14	56	6 18 9	4 10 0	6 23 5
42	24	12	29	7 6 30	3 15 37	8 1 48
43	13	21	18	6 25 46	1 25 23	4 9 51
44	2	6	6	6 15 2	0 15 13	8 17 54
45	21	3	39	7 3 24	11 10 50	9 26 37
46	10	12	27	6 22 39	9 20 38	10 4 40
47	29	10	10	7 11 2	8 26 16	11 13 23
48	17	18	48	7 0 17	7 6 4	11 21 25
49	7	3	37	6 19 33	5 15 52	11 29 28
1750	26	1	10	7 7 55	4 21 20	1 8 11
51	15	9	58	6 27 11	3 1 17	1 16 14
52	3	18	47	6 16 27	1 11 5	1 24 17
53	22	16	19	7 4 49	0 16 42	3 3 0
54	12	1	8	6 24 4	10 26 30	3 11 2
55	1	9	56	6 13 21	9 6 18	3 19 5
56	19	7	29	7 1 43	8 11 50	4 27 48
57	8	16	17	6 20 59	6 21 43	5 5 51
58	27	13	50	7 9 21	5 27 20	6 14 34
59	16	22	39	6 28 36	4 7 8	6 22 37
1760	5	7	27	6 17 52	2 16 50	7 0 39
61	24	5	0	7 6 14	1 22 33	8 9 22
62	13	13	48	6 25 30	0 2 21	8 17 25
63	2	22	36	6 14 46	10 12 9	8 25 28
64	20	20	9	7 3 8	9 17 46	10 4 11
65	10	4	58	6 22 24	7 27 35	10 12 14
66	29	2	30	7 10 46	7 3 12	10 20 57

TABLE I. *concluded.*

Old Stile.

Julian Years.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	H.	M.	S.	S.	°	'	S.	°	'	S.	°	'
1767	18	11	19	7	0	2	5	13	0	11	28	59
68	6	20	8	6	19	18	3	22	48	0	7	2
69	25	17	40	7	7	40	2	28	25	1	15	45
1770	15	2	29	6	26	56	1	8	13	1	23	48
71	4	11	17	6	16	11	11	18	3	2	1	51
72	22	8	50	7	4	33	10	23	38	3	10	34
73	11	17	38	6	23	49	9	3	26	3	18	36
74	1	2	27	6	13	5	7	13	14	3	26	39
75	20	0	0	7	1	27	6	18	51	5	5	22
76	8	8	48	6	20	43	4	28	39	5	13	25
77	27	6	21	7	9	5	4	4	16	6	22	8
78	16	15	9	6	28	21	2	14	4	7	0	11
79	5	23	58	6	17	37	0	23	52	7	8	13
1780	23	21	30	7	5	59	11	29	29	8	16	56
81	13	6	19	6	25	15	10	9	17	8	24	59
82	2	15	7	6	14	30	8	19	5	9	3	2
83	21	12	40	7	2	52	7	24	42	10	11	45
84	9	21	29	6	22	8	6	4	31	10	19	48
85	28	19	1	7	10	30	5	10	8	11	28	31
86	18	3	50	6	29	46	3	19	56	0	6	33
87	7	12	38	6	19	2	1	29	44	0	14	36
88	15	10	11	7	7	24	1	5	21	1	23	19
89	14	18	59	6	26	40	11	15	9	2	1	22
1790	4	3	48	6	15	55	9	24	57	2	9	25
91	23	1	21	7	4	18	9	0	34	3	18	8
92	11	10	9	6	23	33	7	10	22	3	26	10
93	0	18	58	6	12	49	5	20	10	4	4	13
94	19	16	30	7	1	11	4	25	47	5	12	56
95	9	1	19	6	20	27	3	5	35	5	20	59
96	26	22	51	7	8	49	2	11	12	6	29	42
97	16	7	40	6	28	5	0	21	0	7	7	45
98	5	16	29	6	17	21	11	0	48	7	15	47
99	24	14	1	7	5	43	10	6	25	8	24	30
1800	12	22	50	6	24	59	8	16	14	9	2	33

TABLE II.

*Mean New Moon in January, New
Stile.*

Greg. Years.	New Moon.		Sun's Anomaly.		Moon's Anomaly.		Sun from Node.	
	D.	H. M.	S.	°	S.	°	S.	°
1752	14	18.47	6	16	27	1	11	5
53	4	3 38	6	5	43	11	20	53
54	23	1 8	6	24	5	10	26	30
55	12	9 56	6	13	21	9	6	18
5	30	7 29	7	1	43	8	11	55
57	19	16 17	6	20	59	6	21	43
58	9	1 6	6	10	11	5	1	31
59	27	22 39	6	28	36	4	7	8
1760	16	7 2	6	17	52	2	16	56
6	5	16 16	6	7	8	0	26	44
62	24	13 48	6	25	30	0	2	21
63	13	22 37	6	14	46	10	12	9
64	2	7 28	6	4	2	8	21	57
65	21	4 58	6	22	24	7	27	35
66	10	13 46	6	11	40	6	7	23
67	29	11 19	7	0	2	5	13	0
68	17	20 8	6	19	18	3	22	48
69	7	4 56	6	8	53	2	2	36
1770	25	2 49	6	26	56	1	8	13
71	15	11 17	6	16	11	11	18	1
72	3	20 6	6	5	27	9	27	49
73	22	17 38	6	23	49	9	3	26
74	12	2 27	6	13	5	7	13	14
75	1	11 15	6	2	21	5	23	2
76	19	8 48	6	20	43	4	28	39
77	8	17 37	6	9	59	3	8	27
78	27	15 9	6	28	21	2	14	4
79	16	23 58	6	17	37	0	23	52
1780	5	8 46	6	6	52	11	3	40
81	24	6 19	6	25	15	10	9	17
82	13	15 7	6	14	30	8	19	5
83	2	23 56	6	3	46	6	28	53
84	20	21 29	6	22	8	6	4	31
85	10	6 17	6	11	24	4	14	19
86	29	3 50	6	29	46	3	19	56

TABLE II. *concluded. New Style.*

Greg. Years.	New Moon.	Sun's Anomaly.	Moon's Anomaly.	Sun from Node.
	D. H. M.	S. ° ' "	S. ° ' "	S. ° ' "
1787	18 12 38	6 19 2	1 29 44	0 14 36
88	6 21 26	6 8 18	0 9 32	0 22 39
89	25 18 59	6 26 40	11 15 9	2 1 22
1790	15 3 48	6 15 56	9 24 57	2 9 25
91	4 12 37	6 5 11	8 4 45	2 17 27
92	22 10 9	6 23 33	7 10 22	3 26 10
93	11 18 58	6 12 49	5 20 10	4 4 13
94	1 3 46	6 2 5	3 29 58	4 12 16
95	20 1 19	6 20 27	3 5 35	5 20 59
96	8 10 7	6 9 43	1 15 23	5 29 2
97	27 7 40	6 28 5	0 21 0	7 7 45
98	16 16 29	6 17 21	11 0 48	7 15 47
99	6 1 17	6 6 37	9 10 37	7 23 50
1800	24 22 50	6 24 59	8 16 13	9 2 33

TABLE III.

Containing $13\frac{1}{2}$ mean Lunations.

Lun.	New Moon.	Sun's Anomaly.	Moon's Anomaly.	Sun from Node.
	D. H. M.	S. ° ' "	S. ° ' "	S. ° ' "
1	29 12 44	0 29 6	0 25 49	1 0 40
2	59 1 28	1 28 13	1 21 38	2 1 20
3	88 14 12	2 27 19	2 17 27	3 2 1
4	118 2 56	3 26 25	3 13 16	4 2 41
5	147 15 40	4 25 32	4 9 5	5 3 21
6	177 4 24	5 24 38	5 4 54	6 4 1
7	206 17 8	6 23 44	6 0 43	7 4 42
8	236 5 42	7 22 51	6 26 32	8 5 22
9	265 18 36	8 21 57	7 22 21	9 6 2
10	295 7 20	9 21 3	8 18 10	10 6 42
11	324 20 5	10 20 10	9 13 59	11 7 23
12	354 8 49	11 19 16	10 9 48	0 8 3
13	383 21 33	0 18 22	11 5 37	1 8 43
$13\frac{1}{2}$	17 18 22	0 14 33	6 12 54	0 15 20

TABLE IV.

Supplemental to Table I. for finding the mean time of New Moon in January, for 6000 years before or after any given year of the 18th Century, according to the Old Stile.

Centuries. Julian	New Moon.	Sun's Anomaly.	Moon's Anomaly.	Sun from Node.
	D. H. M.	S. ° '	S. ° '	S. ° '
100	4 8 5	0 3 15	8 15 19	4 19 24
200	8 16 10	0 6 29	5 0 37	9 8 48
300	13 0 14	0 9 44	1 15 56	1 28 12
400	17 8 19	0 12 59	10 1 15	6 17 36
500	21 16 24	0 16 13	6 16 33	11 7 0
600	26 0 29	0 19 28	3 1 52	3 26 24
700	0 19 50	11 23 36	10 21 22	7 15 8
800	5 3 55	11 26 51	7 6 40	0 4 31
900	9 11 59	0 0 16	3 21 59	4 23 55
1000	13 20 4	0 3 20	0 7 18	9 13 19
1100	18 4 9	0 6 35	3 22 37	2 2 43
1200	22 12 14	0 9 49	5 7 55	6 22 7
1300	26 20 19	0 13 4	1 23 14	11 11 31
1400	1 15 40	11 17 2	9 12 44	3 0 15
1500	5 53 44	11 20 27	5 28 2	7 19 39
1600	10 7 49	11 23 42	2 13 21	0 9 3
1700	14 15 54	11 26 56	10 28 40	4 28 27
1800	18 23 59	0 0 11	7 13 58	9 17 51
1900	23 8 4	0 3 26	3 29 17	2 7 15
2000	27 16 9	0 6 40	0 14 36	6 26 39
2100	2 11 29	11 10 49	8 4 5	10 15 23
2200	6 19 34	11 14 3	4 19 24	3 4 47
2300	11 3 39	11 17 18	1 4 43	7 24 11
2400	15 11 44	11 20 33	9 20 1	0 13 34
2500	19 19 49	11 23 47	6 5 20	5 2 58
2600	24 3 52	11 27 2	2 20 39	9 22 22
2700	28 11 58	0 0 17	11 5 57	2 11 46
2800	3 7 19	11 4 25	6 25 27	6 0 30
2900	7 15 24	11 7 40	3 10 46	10 19 54
3000	11 23 29	11 10 54	11 26 4	3 9 18

TABLE IV. *concluded.*

Julian Centuries.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	D.	H.	M.	S.	°	'	S.	°	'	S.	°	'
3100	16	7	34	11	14	9	8	11	23	7	28	42
3200	20	15	58	11	17	24	4	26	42	0	18	16
3300	24	23	43	11	20	38	1	12	1	5	7	30
3400	29	7	48	11	23	53	9	27	19	9	26	54
3500	4	3	9	10	28	1	5	16	49	1	15	38
3600	8	11	14	11	1	16	2	2	8	6	5	2
3700	12	19	19	11	4	30	10	17	26	10	24	26
3800	17	3	23	11	7	45	7	2	45	3	13	50
3900	21	11	28	11	11	0	3	18	4	8	3	14
4000	25	19	33	11	14	14	0	3	22	0	22	37
4100	0	14	54	10	18	23	7	22	52	4	11	21
4200	4	22	59	10	21	37	4	8	11	9	0	45
4300	9	7	4	10	24	52	0	23	29	11	20	9
4400	13	15	8	10	28	7	6	8	48	6	9	33
4500	17	23	13	11	1	21	5	24	7	10	28	57
4600	22	7	18	11	4	36	2	9	25	3	18	21
4700	26	15	23	11	7	51	10	24	44	8	7	45
4800	1	10	44	10	11	59	6	14	14	11	26	29
4900	5	18	48	10	15	14	2	29	32	4	15	53
5000	10	2	59	10	18	28	11	14	51	9	5	17
5100	14	10	58	10	21	43	8	0	10	1	24	41
5200	18	19	3	10	24	58	4	15	29	6	14	5
5300	23	3	8	10	28	12	1	0	47	11	3	29
5400	27	11	13	11	1	27	9	16	6	3	22	53
5500	2	6	33	10	5	35	3	15	36	7	11	36
5600	6	14	38	10	8	50	1	20	54	0	1	0
5700	10	22	43	10	12	5	10	16	13	4	20	24
5800	15	6	48	10	15	19	6	21	32	9	9	48
5900	19	14	53	10	18	34	3	6	50	1	29	12
6000	23	22	58	10	21	48	11	22	9	6	18	36

The centurial differences in this Table are equal, but in Lunations themselves they are not.—The following Table shews the centurial variations.

TABLE V.

Variations in the mean times of New and Full Moons for 30 centuries, both before and after the 18th century.

Julian Centuries.	New Moon.			Moon's Anomaly.			Sun from Node.		
	<i>Subtract.</i>			<i>Add.</i>			<i>Subtract.</i>		
	H.	M.	S.	°	'	"	°	'	"
100	0	0	25	0	0	24	0	0	8
200	0	1	40	0	1	36	0	0	32
300	0	3	45	0	3	36	0	1	12
400	0	6	40	0	6	24	0	2	8
500	0	10	25	0	10	0	0	3	20
600	0	15	0	0	14	24	0	4	48
700	0	20	25	0	19	36	0	6	32
800	0	26	40	0	25	36	0	8	32
900	0	33	45	0	32	24	0	10	48
1000	0	41	40	0	40	0	0	13	20
1100	0	50	25	0	48	24	0	16	8
1200	1	0	0	0	57	36	0	19	12
1300	1	10	25	1	7	36	0	22	32
1400	1	21	40	1	18	24	0	26	8
1500	1	33	45	1	30	0	0	30	0
1600	1	46	40	1	42	24	0	34	8
1700	2	0	25	1	55	36	0	38	32
1800	2	15	0	2	9	36	0	43	12
1900	2	30	25	2	24	24	0	48	8
2000	2	46	40	2	40	0	0	53	20
2100	3	3	45	2	56	24	0	58	48
2200	3	21	40	3	13	36	1	4	32
2300	3	40	25	3	31	36	1	10	32
2400	4	0	0	3	50	24	1	16	48
2500	4	20	25	4	10	0	1	23	12
2600	4	41	40	4	30	24	1	30	8
2700	5	3	45	4	51	36	1	37	12
2800	5	26	40	5	13	36	2	44	32
2900	5	50	25	5	36	24	1	52	8
3000	6	15	0	6	0	0	2	0	0

The numbers in this Table are always to be subtracted from the mean time of New Moon, and Sun's mean distance from the Node, as given by the preceding Tables, and added to the mean Anomaly of the Moon, in all Centuries both before and after the 18th.

TABLE VI.

The days in a common year, reckoned from the beginning of January, and serving (with the foregoing Tables) to find the days of New and Full Moons in all the other months.

Days	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

TABLE VII. *First Equation from mean to true Syzygy.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees.	0 Signs.	1 Sign.	2 Signs.	3 Signs.	4 Signs.	5 Signs.	Degrees.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	
0	0 0	2 3	3 35	4 11	3 40	2 8	30
1	0 4	2 7	3 37	4 11	3 37	2 4	29
2	0 9	2 11	3 39	4 11	3 35	2 0	28
3	0 13	2 14	3 41	4 11	3 33	1 56	27
4	0 17	2 18	3 43	4 11	3 30	1 52	26
5	0 21	2 21	3 45	4 10	3 28	1 48	25
6	0 26	2 25	3 47	4 10	3 26	1 41	24
7	0 29	2 28	3 49	4 10	3 23	1 40	23
8	0 34	2 32	3 51	4 9	3 20	1 36	22
9	0 38	2 35	3 52	4 9	3 18	1 32	21
10	0 43	2 39	3 54	4 8	3 15	1 28	20
11	0 47	2 42	3 56	4 7	3 12	1 23	19
12	0 51	2 45	3 57	4 6	3 9	1 19	18
13	0 55	2 48	3 58	4 6	3 6	1 15	17
14	0 59	2 52	4 0	4 5	3 3	1 11	16
15	1 4	2 55	4 1	4 4	3 0	1 6	15
16	1 8	2 58	4 2	4 3	2 57	1 2	14
17	1 12	3 1	4 3	4 1	2 54	0 58	13
18	1 16	3 4	4 4	4 0	2 51	0 53	12
19	1 20	3 7	4 5	3 58	2 47	0 49	11
20	1 24	3 10	4 6	3 57	2 44	0 44	10
21	1 28	3 12	4 7	3 56	2 41	0 40	9
22	1 32	3 15	4 8	3 54	2 37	0 36	8
23	1 36	3 18	4 8	3 53	2 34	0 31	7
24	1 40	3 20	4 9	3 51	2 30	0 27	6
25	1 44	3 23	4 9	3 49	2 26	0 22	5
26	1 48	3 26	4 10	3 48	2 23	0 18	4
27	1 52	3 28	4 10	3 46	2 19	0 13	3
28	1 56	3 30	4 11	3 44	2 15	0 8	2
29	1 59	3 33	4 11	3 42	2 12	0 4	1
30	2 3	3 35	4 11	3 40	2 8	0 0	0
Deg.	11 Signs.	10 Signs.	9 Signs.	8 Signs.	7 Signs.	6 Signs.	Deg.

Add.

TABLE VIII. *Equation of the Moon's mean Anomaly.*

Argument. Sun's mean Anomaly.							
Subtract.							
Degrees.	0 Signs.	1 Sign.	2 Signs.	3 Signs.	4 Signs.	5 Signs.	Degrees.
0	0 0	0 47	1 22	1 35	1 23	0 48	30
1	0 2	0 48	1 22	1 35	1 22	0 47	29
2	0 3	0 50	1 23	1 35	1 21	0 45	28
3	0 5	0 51	1 24	1 35	1 21	0 44	27
4	0 6	0 52	1 25	1 35	1 20	0 42	26
5	0 8	0 54	1 26	1 35	1 19	0 41	25
6	0 10	0 55	1 27	1 34	1 18	0 39	24
7	0 11	0 56	1 27	1 34	1 17	0 38	23
8	0 13	0 58	1 28	1 34	1 16	0 36	22
9	0 15	0 59	1 28	1 34	1 15	0 35	21
10	0 16	1 0	1 29	1 33	1 14	0 33	20
11	0 18	1 2	1 29	1 33	1 13	0 32	19
12	0 19	1 3	1 30	1 33	1 12	0 30	18
13	0 21	1 4	1 30	1 32	1 10	0 28	17
14	0 23	1 5	1 31	1 32	1 9	0 27	16
15	0 24	1 6	1 31	1 32	1 8	0 25	15
16	0 26	1 7	1 32	1 32	1 7	0 23	14
17	0 27	1 9	1 32	1 31	1 6	0 22	13
18	0 29	1 10	1 33	1 31	1 5	0 20	12
19	0 30	1 11	1 33	1 30	1 3	0 18	11
20	0 32	1 12	1 34	1 30	1 2	0 17	10
21	0 33	1 13	1 34	1 29	1 1	0 15	9
22	0 35	1 14	1 34	1 29	0 59	0 13	8
23	0 37	1 15	1 34	1 28	0 58	0 12	7
24	0 38	1 16	1 34	1 28	0 57	0 10	6
25	0 39	1 17	1 34	1 27	0 55	0 8	5
26	0 41	1 18	1 35	1 26	0 54	0 7	4
27	0 42	1 19	1 35	1 26	0 53	0 5	3
28	0 44	1 20	1 35	1 25	0 51	0 3	2
29	0 45	1 21	1 35	1 24	0 50	0 2	1
30	0 47	1 22	1 35	1 23	0 48	0 0	0
1 Deg.	11 Signs.	10 Signs.	9 Signs.	8 Signs.	7 Signs.	6 Signs.	1 Deg.
Add.							

I. *To calculate the true time of New or Full Moon, in any given year and month of the 18th century.*

From Table I. (page 1, 2, 3) write out the mean time of New Moon in January, Old Stile, for the given year, with the mean Anomalies of the Sun and Moon, and the Sun's mean distance from the ascending Node of the Moon's orbit. If you want the time of Full Moon, add the half lunation, with its Anomalies, &c. at the foot of Table III. (page 5) to the foresaid numbers, if the New Moon taken out falls before the 15th of January; but if it falls after, subtract the half lunation, &c. from the said numbers; and write down the respective sums or remainders. If you want to calculate for the New Stile, in any given year from *A. D.* 1752 to 1800, take out the Mean New Moon with its Anomalies, &c. from Table II. (page 4, 5)

In these additions, or subtractions, remember that 60 minutes make a degree,

degree, 30 degrees make a sign, and 12 signs make a circle. And that, when the number of signs you subtract from is less than the number of signs to be subtracted, add 12 signs to the lesser number; and then you will have a remainder to set down. *A sign is marked thus ♊ , a degree thus $^{\circ}$, and a minute thus $'$.*

When the required New or Full Moon is in any given month after January, add as many lunations from Table III. with their Anomalies, &c. to the numbers taken out for January, as the given month is after January; setting them in order below the January numbers: and these added together will give the Mean time of New or Full Moon, with the Anomalies thereto belonging, for the month desired.

With the number of days added together, enter Table VI. (page 9) under the given month; and against that number you have the day of New or Full Moon in the left hand column
(under

(under *Days*) which you are to set before the hours and minutes already found.

But, as will sometimes happen, if the said number of days falls short of any in the column under the given month, add one lunation and its Anomalies to the foresaid sums; and with this new number of days enter Table VI. under the given month, where you are sure to find it the second time, if the first falls short.

Then, with the signs and degrees of the Sun's mean Anomaly, enter Table VII. (pag. 10) and therewith take out the *first Equation from mean to true Syzygy*, making proportions in the Table for the minutes of Anomaly above whole degrees, because the Tables give the Equations only to whole degrees. Subtract this Equation from the mean time of New or Full Moon, if the signs are at the head of the Table, in which case the degrees are in the left hand column and reckoned downward: but if the signs of Anomaly are at the foot of the Table, in
which

which case the degrees thereof are in the right hand column, and reckoned upward, add the Equation to the above found time of New or Full Moon.

With the signs and degrees of the Sun's mean Anomaly enter Table VIII. and therewith take out the *Equation of the Moon's mean Anomaly*; and apply that Equation to the Moon's mean Anomaly, subtracting it therefrom if the signs are at the head of the Table, and their degrees at the left hand; but adding it to the mean Anomaly of the Moon, if the signs of the Sun's Anomaly be at the foot of the Table, and their degrees at the right hand; and you will have the Moon's equated Anomaly; with which enter Table IX. and take out the Equation answering thereto, adding it to the former equated time, if the signs are at the head of the Table, but subtracting it therefrom, if they are at the foot; and the result will give the true time of the required New or Full Moon, near enough for any common Almanack.

The Tables begin the day at Noon, and reckon the hours and minutes thence forward to the noon of the following day. They give the right time in all the months of common years, and in all the months after February in Leap years. But in January and February, in Leap years, a day must be added to the time given by the Tables.

EXAMPLE I.

For the true time of Full Moon in March 1764, New Style.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° ' "	S. ° ' "	S. ° ' "
To Jan. 1764	2 7 25	6 4 2	8 21 57	9 3 31
add $\frac{1}{2}$ Lun.	14 18 22	0 14 33	6 12 54	9 15 20
2 Lunations.	59 1 28	1 28 13	1 21 38	2 1 20
Full \cdot Mar.	17 3 15	8 16 48	4 26 29	11 20 11
First Equ.	+4 6	Ar. 1 Eq.	+1 33	
Second Equ.	17 7 21		4 28 2	
	+4 50		Ar. 2 Eq.	
True time	17 12 11			

D

By

By this short process it appears, that the true time of the required Full Moon was the 17th of March, at 11 minutes past 12 o'clock at night. A few more examples will make the whole matter plain.

EXAMPLE II.

For the true time of New Moon in April, 1764, New Stile.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° '	S. ° '	S. ° '
Jan. 1764 + 3 Lun.	2 7 25 88 14 12	6 4 2 2 27 19	8 21 57 2. 17 27	9 3 31 3 2 1
March First Equ.	31 21 37 + 4 11	9 1 21 Ar. 1 Eq.	11 9 24 + 1 35	0 5 32
Sec. Equ.	32 1 48 - 3 25		11 10 59 Ar. 2. Eq.	
True time	31 22 23			

This shews the true time to be at 22 hours, 23 minutes, after the noon of the 31st of March; which is the 1st of April at 23 minutes past X in the morning.

II. To

II. *To calculate the true time of New or Full Moon, in any given year and month, of any century, between the Christian Æra and the 18th Century. Old Stile.*

In Table I. find a year in the 18th Century, of the same number with that in the Century proposed, and take out the numbers belonging thereto as in the preceding Examples. Then, from Table IV. take out the numbers answering to the number of Centuries before the 18th, subtracting them from those of the 18th, and setting down the remainder.

To this remainder join the numbers for as many Centuries, from Table V. subtracting those for the New Moon, and Sun's distance from the Node, from the said remainder, and adding those for the Moon's Anomaly to it; and the result will give the mean time of New Moon in January, the year of the Century proposed: which being found,

D 2 work,

work, in all respects, for the true time of New or Full Moon in January or any other month of that year, as already shewn.

N. B. If the days annexed to the Centuries taken out from Table IV. exceed the number of days from the beginning of January, taken out in the 18th Century, add a Lunation and its Anomalies, &c. from Table III. to those taken out from the 18th Century; and then you can make a subtraction.

In all calculations for New or Full Moon, either before or after the 18th Century, the variation numbers answering to the Centuries in Table V. must be subtracted from the mean time of New Moon, and from the Sun's mean distance from the Node; and added to the Moon's mean Anomaly, as found for the given time, by the preceding Tables.

EXAMPLE

EXAMPLE III.

*For the true time of New Moon in April,
A. D. 237.*

From *A. D.* 1737 subtract 15 Centuries (*viz.* 1500 years) and there will remain 237.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° '	S. ° '	S. ° '
Jan. 1737	19 14 58	7 1 58	10 4 59	4 20 14
—1500 Years	5 23 44	11 20 27	5 28 2	7 19 39
Remains	13 15 14	7 11 31	4 6 57	9 0 35
Var. 1500 Y.	—1 34	— — —	+1 30	—0 30
Jan. A. D. 237	13 13 4	7 11 31	4 8 27	9 0 5
+ 3 lun	88 14 12	2 27 19	2 17 27	3 2 1
April First Equ.	12 3 52	10 8 50	0 25 54	0 2 6
	+3 15	Ar. 1 Eq.	+1 13	
Sec. Equ.	12 7 5		6 27 7	
	—4 10		Ar. 2 Eq.	
True time	12 2 55			

Hence, the true time required is April 12, at 55 minutes past 12 in the Afternoon.

III. To

III. *To calculate the true time of New or Full Moon in any given year and month before the Christian Æra; Old Stile.*

Find a year in the 18th Century, Old Stile, which being added to the given number of years before Christ, diminished by one, shall make a compleat number of Centuries.

Find this number of Centuries in Table IV. and subtract the numbers belonging to them from those for January, in the 18th Century; and to the remainders join the variations for the like number of Centuries from Table V. and then proceed, as above taught, in applying the Equations to gain the true time required.

The Moon's motion in her Orbit being now quicker than it was in former ages, is the reason for our giving the fifth Table, answering to her accelerations.

EXAMPLE

E

EXAMPLE IV.

*For the true time of New Moon in May,
Old Stile, the year before Christ 585.*

The years 584, added to 1716,
make 2300 years, or 20 complete
Centuries.

By the Precepts.	New Moon	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° ' "	S. ° ' "	S. ° ' "
Jan. 1716	11 16 6	6 24 25	2 12 38	2 25 54
—2300 Years	11 3 39	11 17 18	1 4 43	7 24 11
Remains	0 12 27	7 7 17	1 7 55	7 1 43
Var. 2300 Y.	—3 40	— — —	+3 32	—1 11
B. Chr. 585	0 8 47	7 7 17	1 11 27	7 0 32
+ 5 Lun.	147 15 40	4 25 32	4 9 5	5 3 21
May First Equ.	28 0 27 — 12	0 2 49 Ar. 1 Eq.	5 20 32 — 5	0 3 53
Sec. Equ.	28 0 15 + 1 31		5 20 27 Ar. 2 Eq.	
Tr. time May	28 1 46			

So the true time was May 28th, at
46 minutes past 1 o'clock in the Af-
ternoon.

IV. To calculate the true time of New or Full Moon in any year and month after the 18th Century, in the old Style.

Find a year of the same number in the 18th Century with that of the Century proposed, and take out the New Moon and Anomalies for January, from Table I. for the said year in the 18th Century: then from Table IV. take out the numbers for the Centuries after the 18th, adding them to those of the 18th; to which join the Centurial variations, and then proceed for the true time of New or Full Moon as shewn in the former Precepts.]

[illegible]

EXAMPLE

EXAMPLE V.

*For the true time of Full Moon in April,
Old Stile, A. D. 1903.*

To A. D. 1703 add 200 years, and
the sum will be A. D. 1903.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S ° '	S ° '	S ° '
Jan. 1703. + $\frac{1}{2}$ Lun.	6 5 55 14 18 22	6 18 38 0 14 33	7 26 8 6 12 54	6 7 57 0 45 20
Full \mathcal{D} Jan. + 200 Years	21 0 17 8 16 10	7 3 11 0 6 29	2 9 2 5 0 37	6 23 17 9 8 48
Var. 200 Years	29 16 27 — 2	7 9 40 — — —	7 9 39 + 2	4 2 5 — 1
January + 3 Lun.	29 16 25 88 14 12	7 9 40 2 27 19	7 9 41 2 17 27	4 2 4 3 2 1
April 1903 First Equ.	28 6 37 + 3 18	10 6 59 Ar. 1 Eq.	9 27 8 + 1 15	7 4 5
Second Equ.	28 9 55 — 8 55		9 28 23 Ar. 2 Eq.	
Tr. time, Apr.	28 1 0			

Thus the true time is found to be
April 28th, at 1 o'clock in the Af-
ternoon.

E In

In calculating forward from A. D. 1800, the easiest way is to keep by the Old Stile, and then reduce it to the New, by adding the days difference of Stiles, which will be 12, from A. D. 1800 to 1900.

If the Old and New Stiles had existed from the beginning, there would have been no difference between them in A. D. 200. But from that time forward there would to the end of the world. And, in order to find always how many days (from the 200th year after Christ's birth) must be added to the Old Stile to reduce it to the New, in any given Century, observe the following rule.

Divide the number of the given Century by 4, and (without regarding the remainder, when there is any) add 3 to the quotient; then subtract the sum from the number of the century, and the remainder will be the number of days sought.

Thus, for the 18th Century: which began with A. D. 1701, and will end

with A. D. 1800; the fourth part of 18 (omitting fractions) is 4, which added to 3 makes 7; and 7 being subtracted from 18, leaves 11 remaining, for the number of days between the Old and New Stile.

Again, for the 19th Century, which will begin with A. D. 1801, and end with A. D. 1900, a fourth part of 19 (without regarding fractions) is 4, which being added to 3 makes 7; and 7 being subtracted from 19, leaves 12 remaining for the number of days that must be added to the Old Stile to reduce it to the New, from A. D. 1800 to 1900; and so on.

When it appears, by such as the foregoing calculations of New and Full Moons, that the Sun's distance from the [Ascending] Node of the Moon's Orbit is less than 0 Signs 18 degrees, or more than 5 Signs 12 degrees, so as not to exceed 6 Signs 18 degrees; or when it is more than 11 Signs 12 degrees, at the time of New Moon, the Sun will be eclipsed at that time. And

when the Sun's distance from the Node is less than 0 Signs 12 degrees, or any thing between 5 Signs 18 degrees, and 6 Signs 12 degrees; or more than 11 Signs 18 degrees, at the time of Full Moon, the Moon will be eclipsed at that time. On these principles it appears that there must be Eclipses at the times mentioned in all the preceding Examples except the last. The reason of this is, that the Descending Node of the Moon's Orbit is directly opposite to the Ascending; that is, they are just 6 Signs from each other. And when the Sun is within 18 degrees of either of the Nodes at the time of New Moon, the Sun will be eclipsed at that time. And when the Sun is within 12 degrees of either of the Nodes at the time of Full Moon, the Moon will then be eclipsed.

Because most people are satisfied with knowing on what days of the months the Moon is New and Full, without regarding the time of the day, I shall here give a Table of all the
 days

days of the months on which the mean changes of the Moon fall, from A. D. 1752 to 1800, in the New Stile. The days of Full Moons are then easily found; for when the Change happens before the 15th day of the month, 15 days added to the day of change, will give the day of full Moon; and when the Change is after the 15th day of the month, 15 days subtracted therefrom will give the day of Full Moon.

Within the above limits, the day of any month on which the Moon changeth, in any given year, is found under that month, and right against the year. Thus, suppose it was required to find on what day of March the Change happens in A. D. 1767: under March at the head of the Table, and against 1767 at the left hand is 30; the day of the Change required.

Where the figures are double, as $\frac{1}{30}$ or $\frac{1}{31}$, against any year, and under any month; they shew that the Moon changes on the 1st day of that month, and also on the 30th or 31st thereof.

A Table

A Table shewing on what days of the months the mean Changes of the Moon fall, from A. D. 1752, to A. D. 1800. New Stile.

Years.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1752	16	14	15	13	13	12	11	10	8	8	6	6
1753	4	3	4	3	2	1	30	28	27	26	25	25
						31						
1754	23	22	23	22	21	20	19	18	16	16	14	14
1755	12	11	12	11	11	10	9	7	6	5	4	3
1756	1	—	1	29	28	27	26	25	24	23	22	21
	31		30									
1757	20	18	20	18	18	16	16	14	13	12	11	10
1758	9	8	9	8	7	6	5	4	2	2	30	29
									31			
1759	28	26	28	27	26	25	24	23	21	21	19	19
1760	17	16	16	15	14	13	12	11	9	9	7	7
1761	6	4	6	4	4	2	2	30	28	28	27	26
						31						
1762	25	23	25	23	23	21	21	19	18	17	16	15
1763	14	12	14	13	12	11	11	9	7	7	5	5
1764	3	2	2	1	30	28	28	27	25	25	23	23
			30									
1765	21	18	21	20	19	18	17	16	14	14	12	12
1766	11	9	11	9	9	7	7	5	4	3	2	1
											31	
1767	29	28	30	28	28	26	26	24	23	22	21	20
1768	19	17	18	16	16	14	14	13	11	11	9	9
1769	7	6	7	6	5	4	3	2	30	29	28	27
							31					
1770	26	25	26	25	24	23	22	21	19	19	17	17
1771	15	14	16	14	14	12	12	10	9	8	7	6
1772	5	3	4	2	2	30	30	28	27	26	24	24
				31								
1773	23	21	23	21	21	19	19	17	16	16	14	14
1774	12	11	12	11	10	9	8	7	5	4	3	3
1775	1	—	2	30	29	28	27	26	24	24	22	22
	31		31									
1776	20	19	19	18	17	16	16	14	13	12	11	10

The Table concluded. New Stile.

Years.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1777	9	7	9	7	7	5	5	3	2	1 31	30	29
1778	28	26	28	26	26	24	24	22	21	20	19	18
1779	17	16	17	16	15	14	13	12	10	10	8	8
1780	6	4	5	4	3	2	2	30	28	28	26	26
1781	24	23	24	23	22	21	20	19	17	17	16	15
1782	14	12	14	12	12	10	10	8	7	6	5	4
1783	3	2	3	2	1	29	29	27	26	25	24	23
1784	22	20	21	19	19	18	17	16	14	14	12	12
1785	10	9	10	9	8	7	6	5	3	3	2	2
1786	29	28	29	28	27	26	25	24	22	22	20	20
1787	18	17	19	17	17	15	15	13	12	11	10	9
1788	8	6	7	5	5	4	3	2	30	29	28	27
1789	26	24	26	24	24	22	22	21	19	19	17	17
1790	15	14	15	14	13	12	11	10	8	8	6	6
1791	5	3	5	3	3	1	1	29	27	27	25	25
1792	23	22	22	21	21	19	19	17	16	15	14	13
1793	12	10	12	10	10	8	8	6	5	5	3	2
1794	1	—	1	29	29	27	27	25	24	23	22	22
1795	31	31	31	31	31	31	31	31	31	31	31	31
1796	20	19	20	19	18	17	16	15	13	13	11	11
1797	9	8	8	7	7	5	5	3	2	1	29	29
1798	27	26	27	26	25	24	23	22	21	20	19	18
1799	17	15	17	15	15	13	13	11	10	9	8	8
1800	6	5	6	5	4	3	2	1	29	28	27	26
1800	25	23	25	23	22	21	20	19	17	17	15	15

This Table begins the day at mid-
night, which is according to the com-
mon way of reckoning.

Look

Look for the given year in the left hand column, and against it under the given month you have the day of mean New Moon in that month.

Of the causes and times of Eclipses.

An Eclipse of the Sun is caused by the Moons opaque body passing between the Sun and those parts of the earth from which she hides the whole or part of the Sun: and this can never happen but at the time of New Moon.

An Eclipse of the Moon is caused by the whole or part of her body passing through the earth's shadow: which can never happen but when the Moon is full.

If the Moon's Orbit lay in the plane of the Ecliptic (in which the Earth always moves, and the Sun appears to move) the Sun would be eclipsed at the time of every New Moon; and the Moon would be eclipsed at the time of every Full.

But

But one half of the Moon's Orbit lies on the North side of the Ecliptic, and the other half on the South side of it. Therefore the Moon's Orbit intersects the Ecliptic only in two opposite points, which are called the *Moon's Nodes*; and the angle which the Moon's Orbit makes with the Ecliptic is $5^{\circ} 18'$. The intersection from which the Moon ascends Northward from the Ecliptic is called the Moon's *Ascending Node*; and the opposite intersection, from which the Moon descends Southward from the Ecliptic is called the Moons *Descending Node*. These Nodes move backward in the Ecliptic $19\frac{1}{3}$ degrees every year, from the consequent toward the antecedent signs, and therefore they go quite round the Ecliptic, in 18 years, 225 days, and 5 hours.

From the time of the Sun's being in conjunction with either of the Moon's Nodes, to the time of his being in conjunction with the other, is about $173\frac{1}{2}$ days, at a mean rate; within which

F number

number of days the Eclipses must always happen, in different times of the year. *The days of these Conjunctions are shewn in the following Table, from A.D. 1752 to 1800, N. S.*

Mean Conjunctions of the Sun and Nodes.					
Years.	Afc. Node.	Desc. Node.	Years.	Afc. Node.	Desc. Node.
	Mon. D.	Mon. D.		Mon. D.	Mon. D.
1752	Nov. 5	May 16	1777	July 10	Jan. 17
1753	Oct. 18	Apr. 28			Dec. 30
1754	Sept. 29	Apr. 9	1778	June 21	Dec. 11
1755	Sept. 11	Mar. 21	1779	June 2	Nov. 23
1756	Aug. 23	Mar. 2	1780	May 14	Nov. 4
1757	Aug. 5	Feb. 12	1781	Apr. 26	Oct. 16
1758	July 17	Jan. 25	1782	Apr. 8	Sept. 28
1759	June 29	Jan. 6	1783	Mar. 20	Sept. 10
		Dec. 18	1784	Mar. 1	Aug. 22
1760	June 10	Nov. 30	1785	Feb. 10	Aug. 1
1761	May 22	Nov. 11	1786	Jan. 22	July 15
1762	May 4	Oct. 24	1787	Jan. 4	June 27
1763	Apr. 15	Oct. 6		Dec. 16	
1764	Mar. 27	Sept. 17	1788	Nov. 27	June 9
1765	Mar. 8	Aug. 29	1789	Nov. 8	May 20
1766	Feb. 18	Aug. 10	1790	Oct. 21	May 2
1767	Jan. 31	July 23	1791	Oct. 3	Apr. 13
1768	Jan. 12	July 4	1792	Sept. 14	Mar. 25
	Dec. 23		1793	Aug. 27	Mar. 7
1769	Dec. 5	June 15	1794	Aug. 8	Feb. 16
1770	Nov. 17	May 28	1795	July 21	Jan. 28
1771	Oct. 29	May 9	1796	July 2	Jan. 9
1772	Oct. 10	Apr. 20			Dec. 22
1773	Sept. 22	Apr. 2	1797	June 14	Dec. 4
1774	Sept. 4	Mar. 14	1798	May 26	Nov. 15
1775	Aug. 16	Feb. 23	1799	May 8	Oct. 28
1776	July 28	Feb. 4	1800	Apr. 19	Oct. 9

When

When the Moon changes within 18 days before or after the day of the Sun's being in conjunction with either of her Nodes, the Sun will be eclipsed: and when the Moon is full within 12 days before or after the day of the Sun's conjunction with either of the Nodes, the Moon will be eclipsed. At greater distances of the Sun from the Nodes, there can be no Eclipses of these Luminaries.

As the Table contained on page 30 and 31 shews the days on which the mean changes of the Moon happen, and the Moon is always full on the 15th day before or after the change; and the Table on page 34 shews the days on which the Sun is in conjunction with the Moon's Nodes; we may easily find by these Tables on what days of any given year from A. D. 1752 to 1800, the Sun and Moon must be eclipsed. As, for example.

In the year 1766, the Sun is in conjunction with the Moon's Ascending Node on the 18th of February, and

with the Descending Node on the 10th of August. Now, I find by the Table, page 30, and 31, that in the year 1766, the changes of the Moon are on Jan. 11, Feb. 9, March 11, April 9. May 9, June 7, July 7, Aug. 5, Sept. 4, Oct. 3, Nov. 2, Dec. 1 and 31; and consequently, as the change on Feb. 9th is within 18 days of Feb. 18th when the Sun is in conjunction with the Ascending Node, the Sun must be eclipsed at the time of that change. And as the change on August 5 is within 18 days of August 10, when the Sun is in conjunction with the Descending Node of the Moon's Orbit, the Sun must be eclipsed at the time of that change also. But as all the other changes of the Moon in that year are more than 18 days from the times of the conjunction of the Sun and Nodes, there can be no more than the two abovementioned Eclipses of the Sun in the year 1766.

By adding 15 days to all the changes of the Moon, in the same year,
we

we find the days of all the Full Moons to be Jan. 26, Feb. 24, March 26, April 24, May 24, June 22, July 22, August 20, Sept. 19, Oct. 18, Nov. 17, and Dec. 16. But of all these Full Moons, there are only two which happen within 12 days of the conjunctions of the Sun and Nodes; *viz.* those on the 24th of February and 20th of August: and therefore, it is only on these two days of the year 1766, that the Moon can be eclipsed.

And thus we have a very plain and easy method for finding how many Eclipses there must be of the Sun and Moon in any given year, and the days on which they must fall, according to the mean times of New and Full Moons, from *A. D.* 1752 to *A. D.* 1800. But to shew how to calculate the true times and places of Eclipses for different parts of the earth, would swell out this volume far beyond the intended bulk: and therefore, for such calculations and projections, I beg leave to refer the curious reader to my system
of

of Astronomy, printed for Mr. Millar, Bookseller in the Strand, London; to be now had of Mr. Cadell, successor to Mr. Millar, at his shop opposite Catherine Street in the Strand.

The following Table shews the Sun's true place in the Ecliptic, and his declination from the Equator, at the noon of every day of the second year after Leap year, on the meridian of Greenwich. The signs of the Ecliptic are marked in the Table as follows.

Aries ♈, *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐, *Capricornus* ♑, *Aquarius* ♒, and *Pisces* ♓.

A Table of the Sun's Declination is very useful for finding the Latitudes of places on the Earth. And as the method of doing this by the Declination of the Sun is generally known, we have given the following Table for that purpose, to the nearest mean between Leap year, and the first, second, and third year after.

A Table

*A Table shewing the Sun's Place
and Declination.*

Days	January.		February.	
	Sun's Pl.	S.'s Dec.	Sun's Pl.	S.'s Dec.
	S. °	°	S. °	°
1	11	5 S. 23	12	38 S. 17
2	12	6 22 56	13	39 16 45
3	13	8 22 50	14	40 16 28
4	14	9 22 44	15	41 16 10
5	15	10 22 37	16	41 15 52
6	16	11 22 30	17	42 15 33
7	17	12 22 22	18	43 15 15
8	18	13 22 14	19	43 14 56
9	19	14 22 6	20	44 14 36
10	20	16 21 57	21	45 14 17
11	21	17 21 48	22	45 13 57
12	22	18 21 38	23	46 13 37
13	23	19 21 28	24	46 13 17
14	24	20 21 17	25	47 12 57
15	25	21 21 6	26	47 12 37
16	26	22 20 55	27	48 12 16
17	27	24 20 43	28	48 11 55
18	28	25 20 31	29	48 11 34
19	29	26 20 18	✕ 0	49 11 12
20	☾ 0	27 20 5	1	49 10 51
21	1	28 19 52	2	50 10 29
22	2	29 19 39	3	50 10 7
23	3	30 19 24	4	50 9 45
24	4	31 19 11	5	51 9 23
25	5	32 18 55	6	51 9 1
26	6	33 18 40	7	51 9 38
27	7	34 18 25	8	51 8 16
28	8	35 18 9	9	51 8 53
29	9	35 17 53		
30	10	36 17 37		
31	11	37 17 20		

N. signifies North Decl.
and S. South Decl.

To find the Sun's Place and Declination at the Noon of any given day, in the second year after Leap year.

The Table continued.

Days	May.				June.			
	Sun's Pl.		S.'s Dec.		Sun's Pl.		S.'s Dec.	
	S. °	'	°	'	S. °	'	°	'
1	8	10 55	N. 15	8	II	10 44	N. 22	6
2		11 53	15	26	11	41	22	14
3		12 51	15	43	12	39	22	21
4		13 49	16	1	13	36	22	28
5		14 47	16	18	14	34	22	35
6		15 45	16	35	15	31	22	42
7		16 43	16	52	16	28	22	47
8		17 41	17	8	17	26	22	53
9		18 39	17	24	18	23	22	58
10		19 36	17	40	19	20	23	3
11		20 34	17	55	20	18	23	8
12		21 32	18	11	21	15	23	12
13		22 30	18	26	22	12	23	15
14		23 28	18	40	23	9	23	18
15		24 25	18	55	24	7	23	21
16		25 23	19	9	25	4	23	23
17		26 21	19	22	26	1	23	25
18		27 19	19	36	26	58	23	26
19		28 16	19	49	27	56	23	28
20		29 14	20	1	28	53	23	29
21	II	0 11	20	14	29	50	23	29
22		1 9	20	26	30	47	23	29
23		2 7	20	37	1	45	23	28
24		3 4	20	49	2	42	23	27
25		4 2	21	0	3	39	23	26
26		4 59	21	10	4	36	23	24
27		5 57	21	20	5	33	23	22
28		6 54	21	30	6	31	23	19
29		7 52	21	40	7	28	23	16
30		8 49	21	49	8	25	23	13
31		9 47	21	58				

Sun's Place in the Ecliptic, and his Declination as it is then North or South.

By means of this Table, and the Table of the Semidiurnal Arcs of the

G

Sun

The Table continued.

July.					August.					
Days	Sun's Pl.			S.'s Dec.		Sun's Pl.			S.'s Dec.	
	S.	o	'		'	S.	o	'		'
1	5	9	22	N. 23	9	8	58	N. 18	3	
2		10	19	23	5		9	55	17	48
3		11	16	23	0		10	53	17	32
4		12	14	22	55		11	50	17	16
5		13	11	22	50		12	48	17	0
6		14	8	22	44		13	45	16	44
7		15	5	22	38		14	43	16	27
8		16	1	22	31		15	41	16	10
9		16	59	22	24		16	38	15	53
10		17	50	22	17		17	36	15	35
11		18	54	22	9		18	33	15	18
12		19	51	22	1		19	31	15	0
13		20	49	21	52		20	29	14	41
14		21	46	21	43		21	26	14	23
15		22	43	21	34		22	24	14	4
16		23	40	21	24		23	22	13	45
17		24	38	21	14		24	20	13	26
18		25	35	21	4		25	17	13	7
19		26	32	20	53		26	15	12	47
20		27	29	20	42		27	13	12	28
21		28	27	20	30		28	11	12	8
22		29	24	20	19		29	9	11	47
23	8	0	21	20	7	20	0	7	11	27
24		1	19	19	54		1	5	11	7
25		2	16	19	41		2	3	10	46
26		3	13	19	28		3	1	10	25
27		4	11	19	15		3	59	10	4
28		5	8	19	1		4	57	9	43
29		6	6	18	47		5	55	9	22
30		7	3	18	33		6	53	9	0
31		8	0	18	18		7	51	8	38

Sun and Moon, the times of rising and setting of the Sun, on any day of the year, may be found, in all latitudes from 48 degrees to 59 inclusive. The Sun's

The Table continued.

Days	September.		October.	
	Sun's Pl.	S.'s Dec.	Sun's Pl.	S.'s Dec.
	S. ° ' "	° ' "	S. ° ' "	° ' "
1	8 49	N. 8 17	8 8	S. 3 14
2	9 47	7 55	9 7	3 37
3	10 46	7 32	10 7	4 1
4	11 44	7 10	11 6	4 24
5	12 42	6 48	12 5	4 47
6	13 40	6 26	13 4	5 10
7	14 39	6 3	14 4	5 33
8	15 37	5 41	15 3	5 56
9	16 35	5 18	16 3	6 20
10	17 34	4 55	17 2	6 42
11	18 32	4 32	18 1	7 5
12	19 31	4 9	19 1	7 28
13	20 29	3 47	20 0	7 50
14	21 28	3 23	21 0	8 13
15	22 26	3 0	22 0	8 35
16	23 25	2 37	23 0	8 57
17	24 24	2 14	23 59	9 19
18	25 22	1 51	24 59	9 41
19	26 21	1 27	25 59	10 3
20	27 20	1 4	26 58	10 25
21	28 19	0 40	27 58	10 46
22	29 17	0 17	28 58	11 8
23	30 16	S. 0 7	29 58	11 29
24	1 15	0 30	30 58	11 50
25	2 14	0 53	1 58	12 11
26	3 13	1 17	2 58	12 31
27	4 12	1 40	3 58	12 52
28	5 11	2 4	4 58	13 12
29	6 10	2 27	5 58	13 32
30	7 9	2 51	6 58	13 52
31			7 58	14 11

Sun's Declination is also useful for finding the Latitudes of Places ; for which I have given a great variety of Rules in my Book of Lectures on Mechanics,

The Table concluded.

Days	November.				December.			
	Sun's Pl.		S.'s Dec.		Sun's Pl.		S.'s Dec.	
	S.	°	'		S.	°	'	
1	m	8	58	S. 14 31	† 9	16	S. 21	53
2		9	58	14 50	10	17	22	2
3		10	59	15 9	11	18	22	11
4		11	59	15 28	12	19	22	19
5		12	59	15 46	13	20	22	27
6		13	59	16 4	14	22	22	34
7		15	0	16 22	15	23	22	41
8		16	0	16 39	16	24	22	47
9		17	0	16 57	17	25	22	53
10		18	1	17 14	18	26	22	59
11		19	1	17 30	19	27	23	4
12		20	2	17 47	20	28	23	8
13		21	2	18 3	21	29	23	12
14		22	3	18 19	22	30	23	16
15		23	4	18 34	23	31	23	19
16		24	4	18 49	24	32	23	22
17		25	5	19 4	25	33	23	25
18		26	5	19 19	26	34	23	21
19		27	6	19 33	27	35	23	28
20		28	7	19 47	28	36	23	29
21		29	7	20 0	29	37	23	29
22	†	0	8	20 13	30	38	23	29
23		1	9	20 26	1	39	23	28
24		2	10	20 38	2	40	23	27
25		3	11	20 50	3	42	23	26
26		4	11	21 1	4	43	23	24
27		5	12	21 12	5	44	23	21
28		6	13	21 23	6	45	23	19
29		7	14	21 33	7	46	23	15
30		8	15	21 43	8	48	23	11
31					9	49	23	7

Hydrostatics, Pneumatics, and Optics;
 printed for Mr. Cadell, Bookseller in
 the Strand, London.

*The Table of Semi-diurnal Arcs
for shewing the times of rising
and setting of the Sun, and
Moon.*

		Latitude of the Place.					
North Declination of the Sun and Moon.	Declin.	48°		49°		50°	
		Sun	Moon	Sun	Moon	Sun	Moon
	Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1		6 8	6 20	6 8	6 20	6 8	6 20
2		6 12	6 24	6 12	6 24	6 13	6 24
3		6 17	6 29	6 17	6 29	6 18	6 29
4		6 21	6 33	6 21	6 33	6 22	6 34
5		6 25	6 38	6 26	6 38	6 27	6 39
6		6 30	6 43	6 31	6 44	6 32	6 45
7		6 34	6 48	6 36	6 49	6 37	6 50
8		6 39	6 53	6 41	6 54	6 42	6 55
9		6 44	6 57	6 45	6 58	6 47	7 1
10		6 48	7 1	6 50	7 3	6 52	7 7
11		6 53	7 6	6 55	7 9	6 57	7 12
12		6 58	7 12	7 0	7 14	7 2	7 17
13		7 3	7 17	7 5	7 19	7 7	7 21
14		7 8	7 22	7 10	7 24	7 11	7 29
15		7 13	7 27	7 15	7 31	7 18	7 33
16		7 18	7 32	7 21	7 36	7 24	7 39
17		7 23	7 37	7 26	7 41	7 29	7 44
18		7 28	7 43	7 31	7 46	7 35	7 50
19		7 34	7 49	7 37	7 52	7 41	7 56
20		7 39	7 54	7 43	7 58	7 47	8 2
21		7 45	8 0	7 49	8 5	7 53	8 9
22		7 50	8 6	7 55	8 11	7 59	8 15
23		7 56	8 12	8 1	8 17	8 6	8 22
24		7 59	8 15	8 4	8 20	8 9	8 25

*To find the time of Sun-rising and Sun-
setting, on any given day of the year,
in*

*The Table of Semi-diurnal Arcs
continued.*

Declin.		Latitude of the Place.					
		48°		49°		50°	
		Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	
1	5 59	6 11	5 59	6 11	5 59	6 11	
2	5 54	6 6	5 54	6 6	5 54	6 6	
3	5 50	6 1	5 49	6 1	5 49	6 1	
4	5 45	5 57	5 45	5 57	5 44	5 56	
5	5 41	5 53	5 40	5 52	5 39	5 51	
6	5 36	5 48	5 35	5 47	5 35	5 46	
7	5 32	5 44	5 31	5 43	5 30	5 42	
8	5 27	5 38	5 26	5 37	5 25	5 36	
9	5 23	5 33	5 21	5 31	5 20	5 30	
10	5 18	5 28	5 17	5 27	5 15	5 25	
11	5 13	5 23	5 12	5 22	5 10	5 20	
12	5 9	5 19	5 7	5 17	5 5	5 15	
13	5 4	5 14	5 2	5 12	5 0	5 10	
14	4 59	5 9	4 57	5 7	4 54	5 4	
15	4 54	5 4	4 52	5 2	4 49	4 59	
16	4 49	4 58	4 46	4 56	4 45	4 54	
17	4 44	4 53	4 41	4 51	4 38	4 48	
18	4 39	4 48	4 36	4 46	4 33	4 42	
19	4 34	4 43	4 30	4 40	4 27	4 36	
20	4 28	4 37	4 25	4 33	4 21	4 29	
21	4 23	4 31	4 19	4 27	4 15	4 23	
22	4 17	4 25	4 13	4 21	4 9	4 17	
23	4 11	4 19	4 7	4 15	4 3	4 11	
24	4 8	4 16	4 4	4 12	4 0	4 8	

in all Latitudes from 48 to 59 inclusive.

Find the Sun's declination for the given day, by the preceding Table ;
then,

*The Table of Semi-diurnal Arcs
continued.*

Declin.		Latitude of the Place.					
		51°		52°		53°	
		Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 8	6 20	6 9	6 21	6 9	6 21	6 21
2	6 13	6 25	6 14	6 26	6 14	6 26	6 26
3	6 18	6 30	6 19	6 31	6 19	6 31	6 31
4	6 22	6 34	6 24	6 36	6 25	6 37	6 37
5	6 27	6 39	6 29	6 41	6 30	6 42	6 42
6	6 33	6 46	6 34	6 47	6 36	6 49	6 49
7	6 38	6 51	6 40	6 53	6 41	6 54	6 54
8	6 43	6 56	6 45	6 58	6 47	6 59	6 59
9	6 48	7 1	6 50	7 3	6 52	7 5	7 5
10	6 54	7 7	6 56	7 9	6 58	7 11	7 11
11	6 59	7 13	7 1	7 15	7 3	7 17	7 17
12	7 4	7 18	7 7	7 21	7 9	7 23	7 23
13	7 10	7 24	7 12	7 26	7 15	7 29	7 29
14	7 15	7 29	7 18	7 32	7 21	7 35	7 35
15	7 21	7 35	7 23	7 33	7 27	7 41	7 41
16	7 27	7 41	7 30	7 44	7 33	7 47	7 47
17	7 33	7 48	7 36	7 51	7 40	7 54	7 54
18	7 38	7 54	7 42	7 58	7 46	8 1	8 1
19	7 45	8 1	7 49	8 5	7 53	8 9	8 9
20	7 51	8 7	7 55	8 11	8 0	8 16	8 16
21	7 57	8 13	8 2	8 18	8 7	8 23	8 23
22	8 4	8 20	8 9	8 25	8 14	8 30	8 30
23	8 11	8 27	8 16	8 32	8 22	8 37	8 37
$\frac{1}{2}$	8 15	8 31	8 20	8 36	8 26	8 41	8 41

then, in the Table of Semi-diurnal Arcs, under the Latitude of the place, and against the degrees of the Sun's Declination at the left hand (as the Declination is then North or South)
you

*The Table of Semi-diurnal Arcs
continued.*

		Latitude of the Place.					
Declin.	Deg.	51°		52°		53°	
		Sun	Moon	Sun	Moon	Sun	Moon
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
South Declination of the Sun and Moon.	1	5 58	6 9	5 58	6 9	5 58	6 9
	2	5 53	6 4	5 53	6 4	5 53	6 4
	3	5 49	6 0	5 48	5 59	5 48	5 59
	4	5 44	5 55	5 43	5 54	5 42	5 53
	5	5 39	5 50	5 38	5 49	5 37	5 48
	6	5 34	5 45	5 33	5 44	5 31	5 42
	7	5 29	5 40	5 27	5 38	5 26	5 37
	8	5 23	5 33	5 22	5 32	5 21	5 31
	9	5 18	5 28	5 17	5 27	5 16	5 26
	10	5 13	5 23	5 11	5 22	5 10	5 21
	11	5 8	5 18	5 6	5 16	5 4	5 14
	12	5 3	5 13	5 0	5 10	4 58	5 8
	13	4 57	5 7	4 55	5 5	4 52	5 3
	14	4 52	5 2	4 49	4 59	4 47	4 57
	15	4 41	4 56	4 41	4 54	4 41	4 51
	16	4 46	4 51	4 38	4 47	4 34	4 44
	17	4 35	4 45	4 32	4 41	4 28	4 38
	18	4 29	4 39	4 26	4 34	4 22	4 31
	19	4 23	4 32	4 19	4 28	4 15	4 24
	20	4 17	4 26	4 13	4 21	4 9	4 17
	21	4 11	4 19	4 6	4 14	4 2	4 10
	22	4 4	4 12	4 0	4 8	3 55	4 3
	23	3 58	4 2	3 53	4 1	3 47	3 57
	$\frac{1}{2}$	3 55	3 59	3 49	3 57	3 43	3 54

you have the Sun's Semi-diurnal Arc,
or time of his setting, on that day;
which Arc being doubled, gives the
whole length of the day; and being
subtracted from 12, gives the time of
Sun-rising. Thus, suppose the Lati-

*The Table of Semi-diurnal Arcs
continued.*

Declin.		Latitude of the Place.					
		54°		55°		56°	
		Sun	Moon	Sun	Moon	Sun	Moon
Deg.		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
North Declination of the Sun and Moon.	1	6 9	6 21	6 9	6 21	6 10	6 22
	2	6 15	6 26	6 15	6 27	6 16	6 28
	3	6 20	6 32	6 21	6 33	6 22	6 34
	4	6 26	6 38	6 27	6 39	6 28	6 40
	5	6 31	6 44	6 32	6 45	6 34	6 47
	6	6 37	6 51	6 38	6 52	6 40	6 54
	7	6 43	6 57	6 44	6 58	6 46	7 0
	8	6 48	7 2	6 50	7 4	6 52	7 6
	9	6 54	7 8	6 56	7 10	6 58	7 12
	10	7 0	7 14	7 2	7 16	7 5	7 19
	11	7 6	7 20	7 8	7 22	7 11	7 25
	12	7 12	7 26	7 15	7 29	7 18	7 32
	13	7 18	7 32	7 21	7 35	7 24	7 38
	14	7 24	7 37	7 28	7 42	7 31	7 46
	15	7 31	7 45	7 34	7 49	7 39	7 53
	16	7 37	7 52	7 41	7 56	7 45	8 0
	17	7 44	7 59	7 48	8 3	7 52	8 7
	18	7 51	8 7	7 55	8 10	8 0	8 15
	19	7 58	8 14	8 2	8 18	8 7	8 23
	20	8 5	8 21	8 10	8 26	8 15	8 31
	21	8 12	8 28	8 18	8 34	8 24	8 40
	22	8 20	8 36	8 26	8 42	8 32	8 48
	23	8 28	8 44	8 34	8 50	8 41	8 57
	24	8 32	8 49	8 38	8 54	8 46	9 1

tude to be 52 degrees North; and the day to be the 4th of May, when the Sun's Declination is 16 degrees North. Then, under 52 ° at the head of this Table, and against 16 degrees of North
H decli-

*The Table of Semi-diurnal Arcs
continued.*

Declin.		Latitude of the Place.					
		54°		55°		56°	
		Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 58	6 10	5 58	6 10	5 58	6 10	
2	5 53	6 5	5 53	6 5	5 52	6 4	
3	5 47	5 59	5 47	5 59	5 46	5 58	
4	5 42	5 54	5 41	5 53	5 40	5 52	
5	5 36	5 47	5 35	5 46	5 34	5 45	
6	5 30	5 41	5 29	5 40	5 28	5 39	
7	5 25	5 36	5 23	5 34	5 22	5 33	
8	5 19	5 30	5 17	5 28	5 16	5 27	
9	5 13	5 23	5 12	5 22	5 10	5 20	
10	5 8	5 18	5 6	5 17	5 3	5 14	
11	5 2	5 12	4 59	5 10	4 57	5 8	
12	4 56	5 6	4 53	5 3	4 51	5 1	
13	4 50	5 0	4 47	4 56	4 44	4 53	
14	4 44	4 55	4 41	4 50	4 37	4 46	
15	4 37	4 48	4 34	4 43	4 31	4 40	
16	4 31	4 41	4 27	4 36	4 24	4 33	
17	4 23	4 34	4 21	4 29	4 17	4 25	
18	4 18	4 26	4 14	4 22	4 9	4 17	
19	4 11	4 19	4 7	4 15	4 2	4 10	
20	4 4	4 12	3 59	4 7	3 54	4 2	
21	3 57	4 5	3 52	3 59	3 46	3 54	
22	3 50	3 58	3 44	3 52	3 38	3 45	
23	3 42	3 49	3 36	3 44	3 29	3 36	
24	3 38	3 44	3 32	3 40	3 25	3 32	

declination, I find 7 hours 30 minutes to be the Sun's semi-diurnal arc on that day; which being doubled gives 15 hours for the whole length of the day. The said arc shews that the Sun sets

*The Table of Semi-diurnal Arcs
continued.*

Declin. Deg.		Latitude of the Place.					
		57°		58°		59°	
		Sun	Moon	Sun	Moon	Sun	Moon
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
North Declination of the Sun and Moon.	1	6 10	6 22	6 10	6 22	6 11	6 23
	2	6 16	6 28	6 17	6 29	6 17	6 30
	3	6 22	6 34	6 23	6 35	6 24	6 37
	4	6 29	6 41	6 30	6 42	6 31	6 44
	5	6 35	6 48	6 36	6 49	6 38	6 51
	6	6 41	6 55	6 43	6 56	6 44	6 58
	7	6 48	7 2	6 49	7 3	6 51	7 5
	8	6 54	7 8	6 56	7 9	6 58	7 12
	9	7 1	7 15	7 3	7 16	7 5	7 19
	10	7 7	7 21	7 10	7 24	7 13	7 27
	11	7 14	7 28	7 17	7 31	7 20	7 34
	12	7 21	7 35	7 24	7 39	7 27	7 42
	13	7 28	7 42	7 31	7 46	7 35	7 50
	14	7 35	7 50	7 39	7 54	7 43	7 58
	15	7 42	7 57	7 46	8 2	7 51	8 6
	16	7 49	8 4	7 54	8 10	7 59	8 15
	17	7 57	8 12	8 2	8 18	8 7	8 23
	18	8 5	8 21	8 10	8 26	8 16	8 32
	19	8 13	8 29	8 19	8 35	8 25	8 42
	20	8 21	8 38	8 28	8 45	8 35	8 52
	21	8 30	8 47	8 37	8 55	8 45	9 2
	22	8 39	8 56	8 47	9 5	8 55	9 13
	23	8 49	9 6	8 57	9 15	9 6	9 24
	24	8 54	9 11	9 2	9 29	9 11	9 29

sets at 30 minutes after 7 ; and being subtracted from 12, leaves 4 hours 30 minutes, which shews that the Sun rises at 30 minutes after 4 o'clock. In

*The Table of Semi-diurnal Arcs
concluded.*

Declin.		Latitude of the Place.					
		57°		58°		59°	
		Sun	Moon	Sun	Moon	Sun	Moon
Deg		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
South Declination of the Sun and Moon.	1	5 58	6 10	5 58	6 10	5 57	6 9
	2	5 52	6 3	5 51	6 3	5 51	6 2
	3	5 45	5 56	5 45	5 57	5 44	5 55
	4	5 39	5 50	5 38	5 50	5 37	5 48
	5	5 33	5 44	5 32	5 43	5 31	5 42
	6	5 27	5 37	5 25	5 36	5 24	5 35
	7	5 20	5 30	5 19	5 29	5 17	5 28
	8	5 14	5 24	5 12	5 22	5 10	5 20
	9	5 8	5 18	5 5	5 14	5 3	5 13
	10	5 1	5 11	4 59	5 9	4 56	5 6
	11	4 54	5 4	4 52	5 2	4 49	4 59
	12	4 48	4 57	4 45	4 55	4 42	4 52
	13	4 41	4 50	4 38	4 47	4 34	4 44
	14	4 34	4 43	4 30	4 39	4 27	4 37
	15	4 27	4 36	4 23	4 31	4 19	4 30
	16	4 20	4 29	4 15	4 23	4 11	4 20
	17	4 12	4 21	4 8	4 16	4 3	4 11
	18	4 5	4 13	4 0	4 8	3 54	4 2
	19	3 56	4 4	3 51	3 59	3 45	3 54
	20	3 49	3 56	3 43	3 51	3 36	3 44
	21	3 40	3 48	3 34	3 43	3 27	3 34
	22	3 31	3 39	3 24	3 33	3 17	3 23
	23	3 23	3 29	3 15	3 22	3 6	3 12
	$\frac{1}{2}$	3 18	3 24	3 10	3 17	3 1	3 7

this Table, the effect of the refractions is taken into the account.

And, in any intermediate part of a degree of latitude and declination, it is easy to make proportions in the Table,

for the rising and setting of the Sun.

To find the time of the Moon's rising and setting on any given day of the year.

This is a more operose problem than the former one for finding the rising and setting of the Sun. For here we must find the Moon's age, declination, and time of coming to the Meridian, on the given day.

The number of Days elapsed between the last change and the given day, in any current Luration, are the days of the Moon's age in that Luration. To find the Moon's age on any day of the month in a given year, look for the day of the mean change in that year and month,

The Moon's Distance gone from the Sun on all the different days of her age, from Change to Change.

Days.	Signs.	Deg.	Min.
1	0	12	11
2	0	24	23
3	1	6	34
4	1	18	46
5	2	0	57
6	2	13	9
7	2	25	20
8	3	7	32
9	3	19	43
10	4	1	54
11	4	14	6
12	4	26	17
13	5	8	29
14	5	20	40
15	6	2	52
16	6	15	3
17	6	27	15
18	7	9	26
19	7	21	37
20	8	3	49
21	8	16	0
22	8	28	12
23	9	10	23
24	9	22	35
25	10	4	47
26	10	16	58
27	10	20	9
28	11	11	20
29	11	23	32
$\frac{1}{2}$	0	0	0

month, in the Table, page 30 and 31, and then you have the days of the Moon's age by counting the number of days from the preceding change to the given day. Thus, suppose I would know the Moon's age on the 20th of May, in the year 1766; I look downward from May at the top of the Table, till I come right against 1766 in the left hand column, and against that year, under May, I find that the change was on the 9th of May; and as there are 11 between 9 and 20 (taking in the 20) it appears that the 20th of May was the 11th day of the Moon's age, in the year 1766.

Now, we are to find the time of the Moon's Southing (or coming to the Meridian) on the 11th day of her age: and the common rule for this is, to multiply the Moon's age by 4, and divide the product by 5; the quotient will be the hours (reckoned forward from noon) and the remainder, when there is any, multiplied by 12, will be the minutes of the time when the Moon

is on the Meridian. Thus, for the 11th day of the Moon's age, 4 times 11 is 44, which divided by 5 quotes 8 hours, and the remaining 4 being multiplied by 12 gives 48 minutes. So that, on the 11th day of the Moon's age, the Moon comes to the Meridian at 48 minutes past VIII in the evening.

We must now find the Moon's place in the Ecliptic on the abovementioned 20th of May, when she is 11 days old.

By the Table page 53, I find that on the 11th day of the Moon's age, she is 4 signs 14 deg. 6 min. gone from the Sun. And by the Table on page 41, I find the Sun's place in the Ecliptic, on the 20th of May, to be $8^{\circ} 29' 14''$; or in 29 degrees 14 minutes of Taurus, to which add the Moon's distance from the Sun, 4 signs, 14 degrees, 6 minutes, and the reckoning carries it to $\approx 13^{\circ} 20'$, or 13 deg. 20 min. of Libra; which is the Moon's place in the Ecliptic on the

the 20th of May, according to her mean motion.

Having thus found the Moon's place in the Ecliptic, we must next find her declination, which is the same as the Sun's, when in that place of the Ecliptic, namely 5 deg. 10 min. South, when in the 13 degree of Libra (see the Table page 43); so the Moon's Declination, on the 20th of May, when she is 11 days old, is $5^{\circ} 10'$ South.

Now, to find her Semi-diurnal Arc on that day, suppose in the Latitude of 52 degrees, I look in the Table of Semi-diurnal Arcs (page 48) against 5 degrees of South Declination and under 52° of Latitude; where, in the Moon's column, I find her Semi-diurnal Arc to be 5 hours 49 minutes.

Lastly, to the above-found time of the Moon's coming to the Meridian, 8 hours 48 minutes past Noon, add her Semi-diurnal Arc, 5 hours, 49 minutes, and the sum will be the time of the Moon's setting; namely, at 14 hours 37 minutes past the Noon of
the

the 20th of May; which is the 21st of May at 37 minutes after 2 o'clock in the morning. And her semi-diurnal Arc, 5 hours 49 minutes, being subtracted from 8 h. 48 minutes, the time of her Southings, gives 2 hours 59 min. past Noon for the time of her rising.

When the Moon's Semi-diurnal Arc is greater than the time of her coming to the Meridian, add 12 hours to that time, and then make the subtraction, and the remainder will give the time of the Moon's rising; which will be in the morning.

In this process, which may be gone through in two minutes, we have considered the Moon as moving always in the Ecliptic. But she is sometimes 5 degrees on the North side of it, and at other times as far on the South, which will affect the time of her rising and setting about half an hour, on the parallel of London: more as the Latitude is farther North, and less

as it is farther South : but this difference can happen only twice in a lunation.

A Table shewing how much time is contained in any given number of mean Lunations. The Lunation being 29 days, 12 hours, 44 minutes, 3 seconds, 2^{'''}, 58^{iv}, or 29.53059085108 days.

Lun.	Days. Decimals of a day.
1	29.53059085108
2	59.06118170216
3	88.59177255324
4	118.12236340432
5	147.65295425540
6	177.18354510648
7	206.71413595756
8	236.24472680864
9	265.77531765972

Although the Table seems to go no further than nine mean Lunations, yet it will do for any number from

1 Lunation to 900000000000, by removing the decimal point one place forward for tens of Lunations, two places forward for hundreds of Lunations, three places for thousands, four places for tens of thousands; and so on, as in the following Examples. For, if we wanted to know how much time is contained in 10 Lunations, then
suppose

suppose a cypher put to 1 in the first column, to make it 10, and the decimal point in the first line to be put one place forward, it will be 295.3059085108, for the number of days and decimal parts of a day in 10 Lunations. The decimal parts may be reduced to the known parts of an integral day, by the common method of reducing decimals.

EXAMPLE I.

In 10 Lunations, Qu. How much time?

Lun:		Days. Decimals of a day,
10		295.3059085108
		mult. by 24h.

12236340432
6118170216

Hou.	7.3418042592
	mult. by 60m.

Min.	20.5082575520
	mult. by 60s.

Sec.	30.4954531200
	mult. by 60th.

Th.	29.4271872000
-----	---------------

d.	h.	m.	s.	th.
----	----	----	----	-----

Answer 295 7 20 30 29.8

EXAMPLE II.

In 74212 mean Lunations, Qu. How many days, hours, minutes, &c. ?

Lun.	Days.	Decimals of a day.
70000	2	67141.3595756
4000		118122.36340432
200		5900.118170216
10		295.3059085108
2		59 06118170216
<hr/>		
74212	2191524	20824034896
Lun.	Days.	mult. by 24 ^h .
<hr/>		
		83296139584
		41648069792
<hr/>		
Hours	4.99776837504	
	mult. by 60 ^m .	
<hr/>		
Min.	59.86610250240	
	mult. by 60 ^s .	
<hr/>		
Sec.	51.96615014400	
	mult. by 60 th .	
<hr/>		
thirds	57.96900864000	
<hr/>		
y.	d.	h. m. s. th.
Answer 6000	24	4 59 51 57. 969.

By reduction, 2191524 days contain 6000 Julian years and 24 days.

EXAMPLE III.

In 1000000000000 mean Lunations, Qu. How much time ?

Lun.	Days.
1000000000000	2953059085108 Answer.

In

In Example III, the number of cyphers annexed to 1 are equal to the number of decimal parts in the first line of the Table; and therefore the whole of that line becomes a whole number of integral days, without any fraction. So that, in 100,000,000,000 mean Lunations, there are just 2953059085108 days.

It is somewhat remarkable, that every 49th mean New Moon falls but 1 min. 30 sec. 34 thirds short of the same time of the day as before.

EXAMPLE IV.

Lun.	Days. Decimals of a day.
40	1181.2236340432
9	265.77531765972

49	1446.99895170292
Lun.	Days. mult. by 24h.

399580681168
199790340584

Hou. 23.97484087008
mult. by 60m.

Min. 58.49045220480
mult. by 60s.

Sec. 29.42713228800
mult. by 60th.

Thirds. 25.62793728000

Which wants only 1 minute, 30 sec. 34.4th.
of 1447 days.

A Table

A Table shewing how many mean Lunations are contained in any given quantity of time.

Years.	Lun.	Decimals of a Lunation.	Hours.	Lun.	Decimals of a Lunation.	Sec.	Lun.	Decimals of a Lunation.
1		12.368530038627	1		0.0014109662	1		0.0000003919
2		24.737000077255	2		0.0028219345	2		0.0000007838
3		37.105590115882	3		0.0042328987	3		0.0000011758
4		49.474120154510	4		0.0056438649	4		0.0000015677
5		61.842650193137	5		0.0070548312	5		0.0000019597
6		74.211180231765	6		0.0084657974	6		0.0000023516
7		86.579710270392	7		0.0098767637	7		0.0000027435
8		98.948240309020	8		0.0112877299	8		0.0000031355
9		111.316770347647	9		0.0126986962	9		0.0000035274
Days.	Lun.	Decimals of a Lunation.	Min.	Lun.	Decimals of a Lunation.	Th.	Lun.	Decimals of a Lunation.
1		00.033863189760	1		0.0000235161	1		0.0000000065
2		00.067726379520	2		0.0000470322	2		0.0000000131
3		00.101589659280	3		0.0000705483	3		0.0000000196
4		00.135452759040	4		0.0000940644	4		0.0000000262
5		00.169315948800	5		0.0001175805	5		0.0000000327
6		00.203179138560	6		0.0001410966	6		0.0000000392
7		00.237042328320	7		0.0001646127	7		0.0000000457
8		00.270905518080	8		0.0001881288	8		0.0000000522
9		00.304768707840	9		0.0002116449	9		0.0000000587

For tens of Julian years, days, hours, &c. remove the decimal points one place forward; for hundreds, two places; for thousands, three places; for tens of thousands, four places; and so on, as in the following example. It appears by the first line of the above Table,

Table, that in 100000 Julian years (which contain 36525000 days) there are 1236853 mean Lunations, and .0038627, or $\frac{38627}{10000000}$ parts of a Lunation, which small fraction may be neglected.

[In common working, 'tis sufficient to take in only four or five of the decimal figures.]

EXAMPLE V.

In 6000 Julian years, 24 days, 4 hours, 59 minutes, 52 seconds, Qu. How many mean Lunations?

		Lun. Decimals.	
Years	6000	74211.180231765	
Days	{ 20	0.677263795	
	{ 4	0.135452759	
Hours	4	0.005643864	More Examples would be superfluous.
Min.	{ 50	0.001175805	
	{ 9	0.000211645	
Sec.	{ 50	0.000019597	
	{ 2	0.000000784	
Answer,		74212.000000014	

To explain the Phenomena of the Harvest-Moon, by means of a common globe.

Make chalk-marks all round the globe on the Ecliptic, at $12\frac{1}{6}$ degrees from

from each other (beginning at Capricorn) which is equal to the Moon's mean motion from the Sun from day to day, near enough for your purpose. Then elevate the North pole of the globe to the latitude of any place in Europe; suppose London, of which the latitude is $51\frac{1}{2}$ degrees North.

This done, turn the ball of the globe round westward, in the frame thereof; and you will see that different parts of the Ecliptic make very different angles with the horizon, as these parts rise in the East: and therefore, that in equal times, unequal portions of the Ecliptic will rise. About Pisces and Aries seven of the marks will rise in about two hours and an half, measured by the motion of the index on the horary circle; but about the opposite signs, Leo and Virgo, the index will go over eight hours in the time that 7 marks will rise. The intermediate signs will, more or less, partake of these differences, as they are more or less remote from them.

Hence

Hence it is plain, that when the Moon is in Pisces and Aries, the difference of her rising will be no more than two hours and an half in seven days; but in Virgo and Libra it will be eight hours in seven days; and this happens in every lunation.

The Moon is always opposite to the Sun when she is full; and the Sun is never in Virgo and Libra but in our Harvest-months, and therefore the Moon is never full in Pisces and Aries but in these months. And consequently, when the Moon is about her full in harvest, she rises with less difference of time, for a week, than when she is full in any other month of the year.

Here we consider the Moon as moving always in the Ecliptic. But as she moves in an orbit which is inclined to the Ecliptic, her rising when about the full in Harvest will sometimes not differ above an hour and 40 minutes through the whole of 7 days; and, at other times, it will differ three hours and an half, in a week, according to

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the different positions of the Nodes of her orbit in the Ecliptic, in different years.

In our Winter, the Moon is in Pisces and Aries, about the time of her first quarter; and rises about noon: but her rising is not then taken notice of, because the Sun is above the Horizon.

In Spring, the Moon is in Pisces and Aries, about the time of her change; and then, as she gives no light, her rising cannot be perceived.

In Summer, the Moon is in Pisces and Aries about her third quarter; and then, as she rises not till about midnight, her rising passes unobserved; especially as she is so much on the decrease.

But in harvest, Pisces and Aries are opposite to the Sun; and therefore the Moon is full in them at that time, and rises nearly after Sun-set for several evenings together; which makes her rising very conspicuous at that time of the year, as it is so beneficial to the farmers, in affording them an immediate

diate supply of light after the going down of the Sun, when they are reaping the fruits of the earth.

Rules for solving Astronomical Problems by the Logarithmic Tables of Sines and Tangents,

1. *The Sun's Longitude, or distance from the nearest Equinoctial point (viz. the beginning of Aries or Libra) being given; to find his Declination.*

As Radius is to the Sine of the Sun's distance from the nearest Equinoctial point, so is the Sine of his greatest declination ($23^{\circ} 29'$) to the Sine of his declination sought.

2. *The Sun's Declination being given, to find his distance from the nearest Equinoctial point, and consequently his place in the Ecliptic.*

As the Sine of the Sun's greatest declination is to the Sine of his present decli-

declination, so is Radius to the Sine of his distance gone from, or in going toward, the Equinoctial point required.

3. *The Sun's distance from the nearest Equinoctial point being given, to find his right Ascension.*

As Radius is to the Co-sine of the Sun's greatest declination, so is the Tangent of his distance from Aries or Libra, to the Tangent of his right ascension therefrom.

4. *The Latitude of the place, and the Sun's Declination being given, to find his Ascensional difference.*

As to the Co-tangent of the latitude is to the Tangent of the Sun's declination, so is Radius to the Sine of his ascensional difference required.

5. *The*

5. *The Latitude of the place and the Sun's Declination being given, to find his Amplitude, or the number of degrees he rises and sets from the East and West.*

As the Co-sine of the latitude is to the Sine of the Sun's declination, so is the Sine of his distance from Aries or Libra to the Sine of his Amplitude.

6. *The Sun's right Ascension and 'is greatest Declination being given, to find the Angle of the Ecliptic and Meridian.*

As Radius is to the Sine of the Sun's greatest declination, so is the Co-sine of his right ascension to the Co-sine of the angle sought.

7. *The*

7. *The Latitude of the place and the Sun's Declination being given, to find the Sun's Altitude when he is due East or West.*

As the Sine of the latitude is to the Sine of the Sun's declination, so is Radius to the Sine of his altitude when due East or West.

N. B. By this problem, a true meridian line may be drawn in Summer, when the Sun rises before he comes to the East, and passes by the West before he sets. For, if a long upright wire be set in a truly level board, the shadow of the wire will run Westward on the board when the Sun is due East, and Eastward when the Sun is due West; which will be at the instant when his altitude, observed by a quadrant, agrees with what the problem makes it. And then, if two points are marked in the line of the shadow, and a
7 straight

straight line be drawn through them on the board, and this line be crossed at right angles, in any point, by another straight line, that line will be a true meridian line; and if the wire be placed perpendicularly in the intersection of these two lines, the shadow of the wire will cover the meridian line when the Sun is on the meridian of the place. This may be done best of all about the Summer solstice, because the Sun changes his altitude fastest, and his declination slowest, about that time.

8. *The Latitude of the place and the Sun's Declination being given, to find the Sun's Altitude at six o'clock in the Morning or Evening.*

As Radius is to the Sine of the Sun's declination, so is the Sine of the latitude to the Sine of the Sun's altitude at six o'clock.

By this problem, you may know when it is exactly six o'clock by the
Sun,

Sun, and consequently how to place a Sun-dial true at that instant, provided it be done in the Summer-time, when the Sun is above the horizon at fix. For, if you keep watching, and observing the Sun's altitude with a quadrant, when you judge the time to be a little before fix, till you find his altitude agrees with what the problem makes it, you are sure that it is then precisely fix o'clock by the Sun; to which time, set your watch, and then you may set it to the true equal time by a common Equation table, which shews how much the Sun's time is faster or slower than the equal time, every day of the year.

N. B. In all observations of the Sun's altitude, you must subtract the refraction of the Sun's rays by the Atmosphere from the observed altitude; for otherwise you will not have it true. And for this purpose, I shall subjoin a Table of refractions at the end of these problems, to
shew

shew how much less the true altitude is, than the observed altitude; and when it is so much less than the problem gives, as is equal to the quantity of refraction at the time of the observed altitude, you have the altitude true.

And here, with regard to the placing of Sun-dials, I must make an observation, that may perhaps seem a very odd one to most people; which is, that if the Dial be made according to the strict rules of calculation, and be truly set at the instant when the Sun's center is on the Meridian; it will be a minute too fast in the Forenoon, and a minute too slow in the Afternoon, by the shadow of the Stile; for the edge of the shadow that shews the time is even with the Sun's foremost edge all the time before Noon, and even with his hindmost edge all the Afternoon on the Dial. And it is the Sun's center that determines the time in the (supposed) Hour circles off the heaven. And as the Sun is half a degree in breadth, he takes two minutes

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to

to move through a space equal to his breadth; so that there will be two minutes at Noon in which the shadow will have no motion at all on the Dial. Consequently, if the Dial be set true by the Sun in the Forenoon, it will be two minutes too slow in the Afternoon; and if it be set true in the Afternoon, it will be two minutes too fast in the Forenoon.

The only way that I know of to remedy this, is to set every hour and minute division on the Dial one minute nearer XII than the calculation makes it to be.

The Sun moves 15 degrees in one hour, 30 degrees in two, 45 in three, 60 in four, 75 in five hours, and 90 in six, with respect to the equator; but, in an oblique sphere, the motion of the shadow, either on a horizontal or vertical plane, is very different. To find the degrees and minutes of a degree of the hour distances from XII on a horizontal Dial, say, as Radius is to the sine of the latitude of the place
(which

(which is the same as the angle of the stile height) so is the Tangent of 15 degrees, and of 30, and 45, 60, 75, and 90, to the Tangent of the distance (in degrees and minutes) of XI and I, X and II, IX and III, VIII and IV, VII and V, VI and VI, from XII on the Dial.

The same calculation serves for erect South Dials, only using the Co-latitude instead of the latitude, for the hours and height of the stile.

9. *The Latitude of the place and the Sun's Declination being given, to find the Sun's Azimuth from the North at six o'clock.*

As the Co-sine of the latitude is to Radius, so is the Co-tangent of the Sun's declination to the Tangent of his Azimuth from the North at six.

10. *The Sun's Altitude, Declination, and time of the day, being given; to find the Sun's Azimuth from the North at that time.*

As the Co-sine of the Sun's altitude is to the Sine of the time from Noon (converted into degrees), so is the Co-sine of the Sun's declination to the Sine of his Azimuth from the North,

11. *The Latitude of the place and the Sun's Declination being given, to find the time of the Sun's rising and setting.*

Find the ascensional difference by Problem 4. Then, the degrees of the ascensional difference being converted into time, subtract that time from 6 hours, when the Sun is in γ , δ , π , φ , ρ , and ω , and the remainder will be the time of Sun-rising, and added to 6 will be the time of Sun-setting. But, when the

the Sun is in ϵ , m , γ , ν , μ , and κ , the ascensional difference added to 6 hours gives the time of Sun-rising, and subtracted therefrom gives the time of his setting. How to find his Amplitude at rising and setting, is already shewn by the 5th Problem.

12. *The Latitude of the Place and the Sun's Declination being given, to find the Sun's Meridian Altitude.*

Subtract the latitude from 90 degrees, and the remainder will be the Co-latitude. Then, if the latitude and declination be both North or both South, the Sum of the declination and Co-latitude is the Sun's Meridian altitude. But when either of these is North and the other South, their difference is the Meridian altitude.

To find the Sun's Altitude at any time of the day, by the shadow of an upright object on a horizontal Plane.

As the length of the shadow is to the height of the object, so is Radius to the Tangent of the Sun's altitude at the time of observation.

The Latitude of the place, the Sun's Meridian Altitude, and present Altitude, being given; to find the time of the day.

As Radius is to the Co-sine of the Sun's declination, so is the Co-sine of the latitude to a fourth Sine; and, as that fourth Sine is to Radius, so is the difference between the Sun's meridian altitude and his present altitude to the versed Sine of the time from Noon.

The

The intended Astronomical Problems being finished, we now give the promised Table of Refractions. See pag. 72.

<i>A Table showing the Refractions of the Sun, Moon, and Stars, adapted to their Apparent Altitudes.</i>	Appar. Altit.		Refrac- tion.		Ap. Alt.		Refrac- tion.		Ap. Alt.		Refrac- tion.	
	° ' "		" "		° ' "		" "		° ' "		" "	
	—		—		—		—		—		—	
0	0	33	45	21	2	18	56	0	36			
0	15	30	24	22	2	11	57	0	35			
0	30	27	35	23	2	5	58	0	34			
0	45	25	11	24	1	59	59	0	32			
1	0	23	7	25	1	54	60	0	31			
1	15	21	20	26	1	49	61	0	30			
1	30	19	46	27	1	44	62	0	28			
1	45	18	22	28	1	40	63	0	27			
2	0	17	8	29	1	36	64	0	26			
2	30	15	2	30	1	32	65	0	25			
3	0	13	20	31	1	28	66	0	24			
3	30	11	57	32	1	25	67	0	23			
4	0	10	48	33	1	22	68	0	22			
4	30	9	50	34	1	19	69	0	21			
5	0	9	2	35	1	16	70	0	20			
5	30	8	21	36	1	13	71	0	19			
6	0	7	45	37	1	11	72	0	18			
6	30	7	14	38	1	8	73	0	17			
7	0	6	47	39	1	6	74	0	16			
7	30	6	22	40	1	4	75	0	15			
8	0	6	0	41	1	2	76	0	14			
8	30	5	40	42	1	0	77	0	13			
9	0	5	22	43	0	58	78	0	12			
9	30	5	6	44	0	56	79	0	11			
10	0	4	52	45	0	54	80	0	10			
11	0	4	27	46	0	52	81	0	9			
12	0	4	5	47	0	50	82	0	8			
13	0	3	47	48	0	48	83	0	7			
14	0	3	31	49	0	47	84	0	6			
15	0	3	17	50	0	45	85	0	5			
16	0	3	4	51	0	44	86	0	4			
17	0	2	53	52	0	42	87	0	3			
18	0	2	43	53	0	40	88	0	2			
19	0	2	34	54	0	39	89	0	1			
20	0	2	26	55	0	38	90	0	0			

The

*The Description of an Instrument for
 solving many Astronomical Problems;
 finding the Hour-distances from XII
 on horizontal and vertical Dials;
 forming spherical Triangles, and solv-
 ing the Cases depending thereon, &c.*

Mr Mungo Murray, Shipwright, contrived a very useful instrument several years ago, which he calls *The Armillary Trigonometer*: and I had it some months by me in the year 1757. Since that time, he shewed me a paste-board model of an instrument, much of the same sort, but of a much smaller size; which, I believe, he has not yet made, either of wood or metal. And, as it is a thing that deserves well to be known, on account of its great utility, I have made it of wood, as represented in Plate I. The only addition that I have made to Mr. Murray's scheme, is a circular scale of the Sun's declination for the different days of the year, to save the trouble of referring

7 to

to Tables of the Sun's declination in printed books; as it is one of the *data* that must be had in solving most of the following Problems, which are only a few of those that may be solved by it.

The upright circular board *A* is 12 inches in diameter, and one inch in thickness. It stands on the horizontal foot *B*.

On the left hand side of this board is a flat semicircle *C*; which is made of box wood, and is pinned fast to the board *A*.

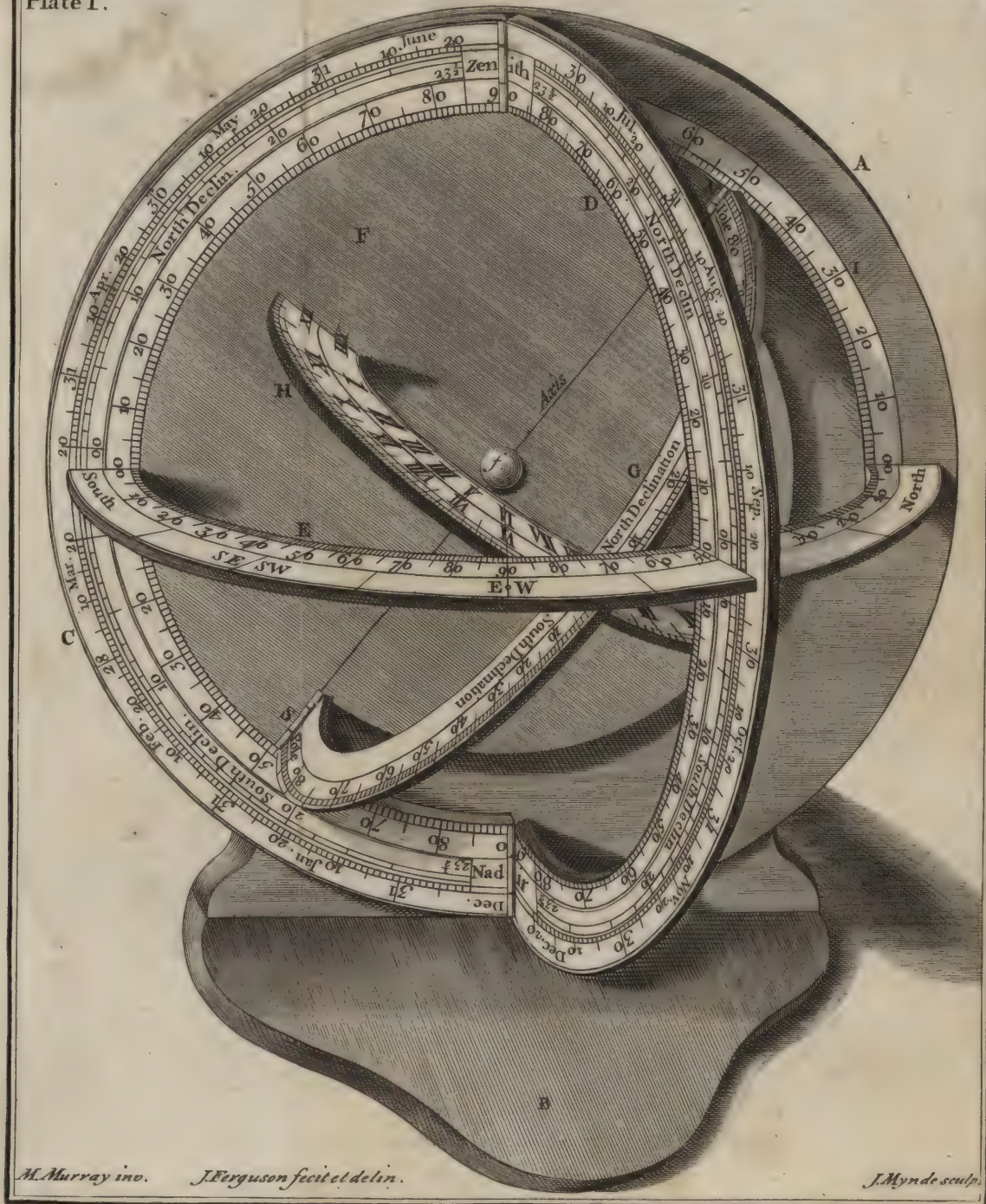
To this semicircle is joined such another, *D*, by two hinges, at the zenith and nadir (so marked in the figure); and is moveable on these hinges. When *D* is put down flat to the board, it and the other make a flat circular ring; on which the months and days of the year are laid down, and all the degrees of the Sun's North and South declinations answering thereto: within which, the four quadrants of the circle are divided into 90 degrees each.

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In the upright board *A*, the ends of the horizontal semicircle *E* are fixed. This semicircle stands at right angles to the plane of the board, parallel to the foot *B*; and is divided forwards and backwards into 32 equal parts, for the points of the compass; within which the two quadrants are divided into 90 degrees each, numbered from the South and North points to the East and West, at *E. W.* where the numbers end at 90.

Within the two semicircles *C* and *D* (when *D* is put down) is the flat board *F*, whose surface lies even with the surfaces of these two semicircles; and which is moveable, round the fixed pin *f* in the center. On this board is a diameter line (marked *Axis*) which represents the axis of the world, and terminates in the North and South poles; where two hinges join the moveable semicircle *G* to the board. This semicircle is divided, upwards and downwards, from the middle to the North and South poles, into twice 90 degrees,
for

Plate I.



M. Murray inv.

J. Ferguson fecit et delin.

J. Mynde sculp.



for all the North and South declinations of the Sun, Moon, and Stars.

In the moveable board *F*, the ends of a semicircle *H* are fixed. The plane of this semicircle is at right angles to the plane of the board, and also to the plane of the semicircle *G* in all positions. It is first divided into twelve equal parts, where the hours are doubly laid down: and then, each hour is subdivided to every fifth minute. The outermost hours are those from Midnight to Noon, and the innermost are the hours from Noon to Midnight.

A quadrant *I*, whose surface is even with the surface of the great board *A*, is divided into 90 degrees, numbered upward from the horizon to the zenith.

As these semicircles answer all the purposes of whole circles, in the instrument; we shall call *D* the *vertical circle*, *E* the *horizon*, *G* the *hour circle*, and *H* the *equator*. There is a notch in *D*, which receives *E*; and the innermost edge of *D* goes close to the outermost edge *G*, whose innermost

edge touches the outermost edge of *H*, in all positions.

The latitude of the place, and the day of the month being given; to rectify the Instrument for use.

In the following Problems, we shall always suppose the latitude of the given place to be North. Therefore, turn the moveable board *F* till the North pole comes to the latitude of the place, on the quadrant *I*; then, find the Sun's declination for the given day of the year, on the semicircles *C* or *D*; and, as that declination is North or South, mark it with chalk, North or South of the equator *H*, on the moveable hour-circle *G*; and the instrument will be rectified: and remember that it must always be so, in each of the following Problems, except the 9th, and 11th.

PROB. I.

PROB. I.

To find the time of the day, either in the Forenoon or Afternoon; and the Sun's true Azimuth from the South at that time.

Observe the Sun's altitude with a quadrant. Then, move the vertical circle *D*, and the hour-circle *G*, till the Sun's observed altitude (above the horizon *E*) on the former, coincides with his declination on the latter: and then, the circle *G* will cut the time of the day in *H* when the observation was made; and the number of degrees reckoned from the South point of the horizon *E* to the vertical circle *D*, will be the Sun's true Azimuth from the South at that time.

PROB. II.

To find the variation of the compass.

The Sun's true azimuth being found by the foregoing Problem, compare it with

with the azimuth shewn by the compass at the time of observation : and the difference will be the variation of the compass at the place where the observation was made.

PROB. III.

The time of the day being given; to find the Sun's altitude and azimuth at that time.

Put the hour circle G to the given time on H ; and, keeping it there, move the vertical circle D till it cuts the Sun's declination in G ; and the intersection will cut the Sun's altitude above the horizon in D , and D will cut the Sun's azimuth from the South in E .

PROB. IV.

To find the time of the Sun's rising and setting, on any day of the year, in any given North latitude less than $66\frac{1}{2}$ degrees.

The reason for confining this Problem within $66\frac{1}{2}$ degrees is, that in
greater

greater latitudes, the Sun continues several natural days (of 24 hours each) above the horizon in summer, without setting; and the time is the longer as the place is the nearer to the pole. At the poles of the earth, the Sun is continually above the horizon for the Summer half year, and continually below it for the Winter half. To solve this limited Problem, turn the hour-circle G till the Sun's declination thereon, for the given day, comes to the horizon E ; and then, G will cut H in the time of the Sun's rising, among the outermost hours; and the time of his setting, among the innermost.

PROB. V.

To find when the Morning Twilight begins, and when the Evening Twilight ends.

When the Sun is just 18 degrees below the horizon in the Morning, the

2

'Twilight

Twilight begins; and when he is 18 degrees below the horizon in the Evening, the Twilight ends. Therefore, mark the 18 degree below the horizon *E* in the vertical circle *D*; and mark the Sun's declination for the given day in the moveable hour-circle *G*. This done, turn *D* and *G* on their hinges till you find the 18th degree below the horizon on *D* cuts the declination on *G*: and then *G* will cut *H* in the time when the Morning Twilight begins, among the outermost hours; and the time when the Evening Twilight ends, among the innermost.

N. B. When the point of the Sun's declination, in the Summer months, does not go so far as 18 degrees below the horizon, at Midnight; the Twilight continues all the Night.

PROB. VI.

A place being given in the North frigid zone (that is, in more than $66\frac{1}{2}$ degrees of North latitude) to find on what day of the year the Sun begins to shine constantly on that place without setting; and how long he continues to do so.

The pole being elevated to the latitude of the place, put down the moveable hour-circle *G* quite flat to the board *A*; and then, observe what degree, or point of North declination on *G*, cuts the horizon *E*. When the Sun is at that point of declination, before the 21st of June, he begins to go on, without setting; and continues to do so till he comes to the like point of declination after the 21st of June. Therefore, the two days, before and after the 21st of June, which answer to the said point of declination in the scale of months, give the solution of the Problem: *that* before the 21st of June
N
being

being the day on which the Sun begins to go round with setting; and he continues to do so, till the *other* after the 21st of June, on which he begins to set; and then to rise and set as at other places.

PROB. VII.

To find how long the Twilight continues at the poles of the Earth.

The continuance is equal at each pole, but at contrary times of the year: suppose therefore we take it for the North pole.

At the North pole, while the Sun is above the horizon, his altitude is equal to his declination North: and while he is below the horizon, his depression is equal to his declination South; and his South declination begins on the 23d of September, and ends on the 20th of March. But, as his South declination is within 18 degrees from the 23d of September to the 13th of
of

of November, the Twilight continues all that time after the Sun sets. And as it is within 18 degrees from the 29th of January to the 20th of March, there is Twilight all that time at the North pole before the Sun rises to it.

PROB. VIII.

To find the Sun's depression below the horizon, at any time of the night, in any given latitude less than $66\frac{1}{2}$ degrees.

The instrument being rectified, bring the moveable circle *G* to the given time of the night in *H*; then, move the vertical circle *D* till it cuts the Sun's declination for the given day in *G*, and the declination in *G* will cut the number of degrees of the Sun's depression below the horizon at that time, in *D*.

PROB. IX.

To find in what North latitude the longest day is of any given length less than 24 hours.

In Northern latitudes, the longest day is when the Sun's declination is $23\frac{1}{2}$ degrees North. Divide the given length by two, and the quotient will give the time of Sun-setting: to which time, place the circle *G* among the innermost hours on *H*; and then, turn the moveable board *F* till $23\frac{1}{2}$ degrees of North declination on *G* comes down to the horizon *E*; and the elevation of the pole above the horizon will shew the latitude of the place, in the quadrant *I*.

PROB. X.

To find the Sun's amplitude at rising and setting, in any given latitude less than $66\frac{1}{2}$ degrees.

The time of the Sun's rising and setting being found by Prob. IV, count the
number

number of degrees on the horizon *E*, at which he rises and sets from the East and West points; and *that* number will be the Sun's amplitude.

PROB. XI.

The length of the longest day being given, at any place whose latitude is North; to find the latitude of that place.

If the given length be less than 24 hours, subtract its half from 12 hours; and the remainder will be the time of Sun setting on that day. To which time, place the moveable hour-circle *G*, among the innermost hours on *H*; and then, turn the moveable board *F* till $23\frac{1}{2}$ degrees of North declination on *G* comes down to the horizon *E*, and the pole will then point out the latitude of the place, in the quadrant *I*: the elevation of the pole above the horizon of any place being always equal to the latitude of that place.

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If the length of the longest day consists of several natural days, of 24 hours each; take the Sun's altitude by a quadrant when he is due North, on the 21st of June; at which time his declination is $23\frac{1}{2}$ degrees North. Then, put down the semicircles *D* and *G*, flat to the board *A*; and turn the moveable board *F* till $23\frac{1}{2}$ degrees of North declination on *G* cuts the Sun's observed altitude on *D*: and then, the North pole will point to the latitude of the place, in the quadrant *I*.

PROB. XII.

In the Summer months, to find an East and West line; and consequently a Meridian line, for a place of any given latitude.

This is best done about the time of the Summer solstice; because the Sun's declination changes slower about that time than any other in Summer: and it cannot be done in the Winter-half
of

of the year, by the instrument; because, during that time, the Sun is always past the East before he rises; and he sets before he comes to the West.

Having set a wire upright in a level board, on which the Sun may shine when he is due East or due West, as already mentioned (pag. 70) and the instrument being rectified, bring the vertical circle *D* to the East and West point of the horizon *E*, and turn the moveable hour-circle *G* till the Sun's declination thereon, for the given day, comes to the vertical circle *D*; and the Declination in *G* will cut the Sun's altitude in *D* when he is due East or due West on that day. For the rest of the operation, see pag. 70 and 71.

PROB. XIII.

To find the distances of all the Forenoon and Afternoon hours from XII, on a horizontal dial for any given Latitude.

Elevate the pole in the quadrant *I*, to the latitude of the given place; and
 3 bring

bring the moveable hour-circle *G* successively to all the outermost hours on the equator *H*; and *G* will cut the distances of all the Forenoon hours from XII on the horizon *E*, as you bring it to the like hours on the equator. The Afternoon hours being at the same distances from XII as the Forenoon hours are, by having the latter we have also the former.

N. B. When $23\frac{1}{2}$ degrees of North declination on *G* comes to the horizon *E*, *G* will cut *H* in the time of the Sun's rising and setting on the longest day; and consequently will limit the number of hours to be put upon the dial. By the same method, the half hour and quarter distances from XII may be found, for all the hours on the dial.

PROB. XIV.

To find the distances of the Forenoon and Afternoon hours from XII, on a vertical South dial for any given latitude.

The pole being elevated in the quadrant *I*, to the latitude of the given place, bring the vertical circle *D* to the East and West point of the horizon *E* (at E. W.) This done, bring the moveable circle *G* successively to all the outermost hours on *H*, from XII to VI; and *G* will cut *D* in all these hour-distances from XII, reckoned downward from the zenith to the horizon. These are the Forenoon hours: and as all the Afternoon hour-distances from XII to VI are the same as the Forenoon hour-distances, 'tis needless to work for them by the instrument.

N. B. On all erect direct South dials, the Forenoon hours begin at VI in the Morning; and the Afternoon

O
hours

hours end at VI in the Evening: for the Sun never shines more than twelve hours on any dial whose plane is perpendicular to the horizon.

The meridian, or twelve o'clock line, on these two dials, must be made as broad as the stile is thick. The angle of the stile's height must be equal to the latitude of the place for which the horizontal dial is made; and the angle of the stile's height in an erect direct South dial must be equal to the Co-latitude of the place.

Having thus found the hour-distances from XII by the instrument, set them off with your compasses, by a line of chords, from the twelve o'clock line on your dial plate; which line being made as broad as the stile is thick, set off the Forenoon hours from the edge of the twelve o'clock line, which is to the Forenoon side of the dial; and the Afternoon hours from the Afternoon edge of that line.

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The fix o'clock line is perpendicular to the meridian line on these dials. It must be drawn before you begin to set off the hour-distances on the dial: and the centers of the two quadrantal arcs (taken equal to the chord of 60 degrees on your scale) must be in those points of the edges of the broad meridian line, where the fix o'clock line intersects it. And the broad edge of the stile, that shews the time by the shadow, must rise from those points in the dial which were made the centers of the above quadrantal arcs whereon the hour-distances are set off from XII.

PROB. XV.

To find the distances of the Forenoon and Afternoon hours from XII, on a vertical dial, declining from the South toward the East or West, by any given number of degrees.

Let us suppose that face of the dial
must decline (or turn away) 30 degrees
O 2 from

from the South toward the East; then, 'tis evident, that the Afternoon edge of the plane of the dial is 30 degrees from the East toward the North, and the Forenoon edge thereof is 30 degrees from the West toward the South.

Therefore, count the 30 degrees of declination from the East point of the horizon *E* (at 90, under E. W.) toward the North point; and where the reckoning ends (*viz.* at 60 degrees from the North), place the vertical circle *D* in the horizon; and *D* will represent the plane of the declining dial. Then, to find the distances of the Forenoon hours from XII on the dial, bring the moveable hour-circle *G* successively to all the Forenoon hours (which are the outermost) on the equator *H*, as XI, X, IX, &c. till *G* comes to the horizon *E* in the point where the vertical circle *D* intersects it. And in doing this, *G* will cut the distances on *D* of all the Forenoon hours from XII, that must be put upon the dial; these distances
being

being reckoned downward on *D*, from the zenith to the horizon.

Then, because we have only semi-circles in the instrument, to find all the hour-distances by; and as the Afternoon hours are not equidistant with the Forenoon hours from XII on declining dials; let the Afternoon hours I, II, III, &c. be reckoned among the outermost on the equator *H* from the right hand toward the left, and let their distances from XII be taken upward on the vertical circle *D*, from the nadir toward the horizon *E*. To find these distances, bring the moveable hour-circle *G* successively to the hours I, II, III, &c. (which are outermost) on the equator, till it comes to *that* point of the horizon *E* where the vertical circle *D* intersects it; and *G* will cut *D* in the distances (reckoned upward from the nadir) of all the Afternoon hours that must be inserted on the dial.

Having thus found the distances from XII, of all the Forenoon and Afternoon hours that must be inserted

on the declining dial, and wrote them down; draw a single line (no thicker than the other hour lines) across the plane of the dial, perpendicular to that edge of it which must be the lowermost, and be parallel to the horizon, when the dial is set; and that line, perpendicular to the horizon, will be the Meridian or twelve o'clock line of the dial.

Near the uppermost end of that line, assume a point for the center of the dial; and, having taken 60 degrees from the line of chords in your compasses, set one foot in the center-point, and with the other foot describe a semicircle on the dial-plane; and thereon set off all the above-found distances from the XII o'clock line; and place the hours at these distances accordingly.

Then, to find the distance of the substile (or line on which the stile must stand) from the meridian line of the dial; bring the moveable hour-circle *G* to as many degrees from the South toward the East point of the horizon *E* as the vertical circle *D* stands at
from

from the East toward the North point; and the circles *D* and *G* will cross each other at right angles. Then, the number of degrees on *D*, which are intercepted between *G* and the zenith, will be the angle that the substile makes with the meridian of the dial; which must be set off from XII, among the Forenoon hours, because the face of the dial declines from the South toward the East. And the number of degrees on *G*, which are intercepted between the vertical circle *D* and the North pole, will be the angle of the stile's height.

The substile line must be drawn to the center point of the dial, that is, to the center of the semicircle on which the hour-distances were set off from XII; and the edge of the stile that shews the hours by the shadow, must begin to rise from the dial at the center point.

In this dial, the stile must be very thin, or else have a sharp edge. If the dial declines Westward from the South, the vertical circle *D* must be
placed

placed as many degrees from the East point of the horizon, toward the South, as the dial declines: and then, the hour-distances from XII are to be found in the same manner as above described.

In East-declining dials, the substile falls among the Forenoon hours; and in West-declining dials, among the Afternoon hours. For, in all kinds of dials, when they are properly set, the edge of the stile, that casts the shadow for shewing the time of the day, must be parallel to the earth's axis.

Every one who reads this, and understands the use of the globes, will easily see that these are only a few of the Problems which may be solved by this instrument. And a bare view of the figure of it is sufficient to shew, that any spherical triangle may be readily formed and solved by it; and consequently, all the Problems that depend on spherical trigonometry. That justice might have been done to it, I wish Mr. Murray himself had described it, and shewn all its uses.

*To know, by the Stars,
whether a clock goes true
or not.*

The stars make 366 revolutions from the Meridian to the Meridian again, or from any point of the compass to the same point again, in 365 days; and therefore they gain a 365th part of a revolution every 24 hours of mean solar time.

Consequently, if you mark the precise moment shewn by a clock, when any star vanishes behind a chimney, or other object, as seen through a small hole in a thin plate of metal, fixed in a window-shutter; and do this for several nights together (as suppose 20) if at the end of that time the star vanishes

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A Table shewing
the daily accelerations
of the Stars.

Days.	Accelerations.			
	Hob.	Min.	Sec.	Th.
1	0	3	55	54
2	0	7	51	48
3	0	11	47	42
4	0	15	43	36
5	0	19	39	30
6	0	23	35	24
7	0	27	31	18
8	0	31	27	12
9	0	35	23	6
10	0	39	19	0
11	0	43	14	54
12	0	47	10	48
13	0	51	6	42
14	0	55	2	36
15	0	58	58	30
16	1	2	54	24
17	1	6	50	18
18	1	10	46	12
19	1	14	42	6
20	1	18	38	0
21	1	22	33	54
22	1	26	29	48
23	1	30	25	42
24	1	34	21	36
25	1	38	17	30
26	1	42	13	24
27	1	46	9	18
28	1	50	5	12
29	1	54	1	6
30	1	57	57	0

as much sooner than it did the first night, by the clock, as answers to the accelerations for so many days in the Table; the clock goes true; otherwise not. If the difference between the clock and star be less than the Table shews, the clock goes too fast; if greater, it goes too slow; and must be regulated accordingly, by letting down or raising up the ball of the pendulum, by little and little, till you find it keep to true equal time.

Thus, suppose the star should disappear behind the chimney any night when it is XII by the clock; and that, on the 20th night afterward, the same star should disappear when it is 41 minutes, 22 seconds, past X by the clock, which subtracted from XII h. 0 m. 0 s. leaves remaining 1 hour, 18 minutes, 38 seconds, for the time the star is then faster than the clock; look in the Table, and against 20, in the left-hand column, you will find the acceleration of the star to be 1 hour, 18 min. 38 seconds; agreeing exactly with
what

what the difference between the clock and star ought to be; which shews that the clock measures true equal time.

Dr. *Desaguliers* informs us, that the length of a pendulum (from the point of suspension to the center of oscillation) that swings seconds in the latitude of London, is 29.128 inches. Now, to find the length of a pendulum that shall make any other given number of vibrations in the same latitude, in a minute; say, as the square of the given number of vibrations is to the square of 60, so is 29.128 inches (the length of the standard) to the length in inches of the pendulum sought.

By this rule, the following Table is calculated, for all numbers of vibrations in a minute, from 1 to 180, serving for the latitude of London. And, by the next Table that follows (page 110) the pendulum may be corrected for any other latitude.

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A Table,

A Table, shewing of what length a Pendulum must be, to make any given number of Vibrations in a minute, from 1 to 180, in Lat. 51° 30'.

Vibrations.	Length of the Pendulum.		Vibrations.	Length of the Pend.		Vibrations.	Length of the Pend.	
	Feet.	Inches.		Feet.	Inches.		Feet.	Inches.
1	11738	4.800	31	12	2.577	61	3	1.852
2	2934	7.200	32	11	5.637	62	3	0.644
3	1304	3.200	33	10	9.358	63	2	11.490
4	733	7.800	34	10	1.852	64	2	10.387
5	469	6.432	35	9	6.998	65	2	9.340
6	325	0.800	36	9	0.681	66	2	8.337
7	239	6.710	37	8	6.894	67	2	7.381
8	183	4.800	38	8	1.543	68	2	6.465
9	144	11.022	39	7	8.611	69	2	5.586
10	116	8.608	40	7	4.038	70	2	4.747
11	97	0.139	41	6	11.736	71	2	3.943
12	80	11.255	42	6	7.753	72	2	3.172
13	69	2.538	43	6	4.074	73	2	2.433
14	59	5.566	44	6	0.759	74	2	1.723
15	52	2.048	45	5	9.561	75	2	1.042
16	45	10.234	46	5	6.569	76	2	0.387
17	40	7.408	47	5	3.767	77	1	11.758
18	35	2.755	48	5	1.137	78	1	11.152
19	32	6.196	49	4	10.607	79	1	10.571
20	29	4.154	50	4	8.344	80	1	10.009
21	26	7.639	51	4	6.156	81	1	9.469
22	24	3.034	52	4	4.093	82	1	8.949
23	22	2.277	53	4	2.146	83	1	8.447
24	20	4.550	54	4	0.169	84	1	7.963
25	18	8.377	55	3	10.465	85	1	7.496
26	17	4.374	56	3	8.818	86	1	7.045
27	16	1.238	57	3	7.355	87	1	6.610
28	14	11.669	58	3	5.873	88	1	6.190
29	13	11.492	59	3	4.466	89	1	5.771
30	12	0.512	60	3	3.128	90	1	5.390

The Pendulum Table concluded.

Vibrations.	Length of the Pend.		Vibrations.	Length of the Pend.		Vibrations.	Length of the Pend.	
	Feet.	Inch.		Inches.			Inches.	
91	1	5.040	121	9.620		151	6.175	
92	1	4.642	122	9.464		152	6.096	
93	1	4.252	123	9.310		153	6.017	
94	1	3.939	124	9.161		154	5.939	
95	1	3.597	125	9.015		155	5.863	
96	1	3.284	126	8.872		156	5.788	
97	1	2.971	127	8.727		157	5.715	
98	1	2.670	128	8.597		158	5.643	
99	1	2.372	129	8.464		159	5.572	
100	1	2.068	130	8.335		160	5.502	
101	1	1.800	131	8.208		161	5.434	
102	1	1.539	132	8.084		162	5.367	
103	1	1.277	133	7.963		163	5.301	
104	1	1.006	134	7.844		164	5.238	
105	1	0.776	135	7.728		165	5.211	
106	1	0.536	136	7.615		166	5.112	
107	1	0.303	137	7.510		167	5.051	
108	1	0.076	138	7.396		168	4.991	
109	0	11.856	139	7.291		169	4.932	
110	0	11.641	140	7.187		170	4.874	
111	0	11.433	141	7.085		171	4.817	
112	0	11.211	142	6.985		172	4.761	
113	0	11.031	143	6.888		173	4.707	
114	0	10.838	144	6.793		174	4.652	
115	0	10.561	145	6.699		175	4.599	
116	0	10.368	146	6.608		176	4.547	
117	0	10.281	147	6.518		177	4.496	
118	0	10.117	148	6.431		178	4.445	
119	0	9.954	149	6.345		179	4.396	
120	0	9.782	150	6.260		180	4.347	

Length for 300 Vibrations
in a min. 1 inch and .565
parts of an inch.

To make the lengths of pendulums answer to the intended numbers of Vibrations, as in this Table; the weight of the ball, in all cases, should bear the same proportion to the weight of the pendulum rod, as the weight of the standard-ball bears to the weight of its rod.

A Table, shewing how much a Pendulum that swings Seconds at the Equator would gain every 24 hours in different Latitudes; and how much the Pendulum would need to be lengthened in these Latitudes, in order to make it swing seconds therein.

Latitude of the Place.	Time gained in one Day.	Lengthening of the Pendulum to swing Seconds.	Latitude of the Place.	Time gained in one Day.	Lengthening of the Pendulum to swing Seconds.
Deg.	Seconds.	Inch. Parts.	Deg.	Seconds.	Inch. Parts.
5	1 .7	0 .0016	50	134 .0	0 .1212
10	6 .9	0 .0062	55	153 .2	0 .1386
15	15 .3	0 .0138	60	171 .2	0 .1549
20	26 .7	0 .0246	65	187 .5	0 .1696
25	40 .8	0 .0369	70	201 .6	0 .1824
30	57 .1	0 .0516	75	213 .0	0 .1927
35	75 .1	0 .0679	80	221 .4	0 .2033
40	94 .3	0 .0853	85	226 .5	0 .2050
45	114 .1	0 .1033	90	228 .3	0 .2065

A pendulum that swings seconds at the equator must be $\frac{128}{1000}$ parts of an inch shorter than one that swings seconds at London; and a pendulum that swings seconds at the poles must be $\frac{80}{1000}$ parts of an inch longer than one that swings seconds at London. The cause of this difference arises from the spheroidal

roidical figure of the earth, and the centrifugal force diminishing (and so acting gradually, less and less), from the equator to the North and South poles, as the diurnal motions of the places are slower and slower.

The length of a pendulum that swings seconds at the equator is 39 inches; and the length of a pendulum that swings seconds at the poles, is 39.266 inches.

A Description of three uncommon kinds of Clocks.

I. About twenty years ago, I made a wooden model of a clock, which shews the apparent diurnal motions of the Sun, Moon, and Stars, with the times of their rising, southing, and setting, for every day of the year; together with all the various phases of the Moon, and times of her being New and Full in the different months of the year; with the days of the months, never needing to be shifted, save once

in four years; and the age of the Moon for every day of the year, and to every third hour from her change, in any current Lunation. All these are shewn on the dial-plate, without any confusion; and I keep the model still by me to shew in my lectures.

The outer part of the dial-plate is divided into twice twelve hours, and each hour into eight equal parts, for the half hours, quarters, and half quarters.

Within this circle of hours there is a ring, which goes round once in 24 hours, and carries an index for pointing to the hours of the day and night, and a gilt ball for representing the Sun, and shewing his apparent diurnal motion round the earth. This ring is divided into 29 days, 12 hours, and 45 minutes, for the Moon's age from change to change. A ball, half black, half white, is turned round its axis in 29 days, 12 hours, and 45 minutes, for shewing all the various phases of the Moon: the axis of the ball lies in the plane of the ring, and comes out a little way beyond

beyond the Moon, and points to her age in the foresaid divisions on the ring, falling back every day as much as the Moon is later of coming to the meridian every day than she was on the day before; and consequently, the Moon falls back every day so far in the ring, as to go round it in 29 days 12 hours 45 minutes, from the Sun to the Sun again.

Within this ring is a flat circular plate, divided all around its edge into 365 equal parts, for the months and days of the year, which are set at the proper divisions. This plate makes 366 revolutions (as the stars do) in the time the Sun makes 365; by which means, the wire that carries the Sun round in 24 hours, cuts the day of the month on the plate, as the plate advances a 365th part of a revolution upon the Sun, once every 24 hours: so that the plate turns round in a syderal day, which consists of 23 hours, 56 m. 4 s. 6 thirds, of mean solar time,

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and the Sun goes round in 24 mean solar hours.

The equator, ecliptic, and tropics are drawn on this plate; the ecliptic is divided into the 12 signs of the Zodiac, and each sign into 30 degrees. The wires, which carry the Sun and Moon, cut their places in the Ecliptic, for every day of the year. All the remarkable stars of the first and second magnitudes are laid down on this plate, according to their right ascensions and declinations.

Over this plate is a fixt horizon for shewing the times of the rising and setting of the Sun, Moon, and Stars. When any star comes to the East side of the horizon, it rises; and the hour-hand points to the time of its rising: when it sets on the Western side of the horizon, the hour-hand points out the time of its setting; on any day of the year which the Sun's wire then cuts.

When the points of the ecliptic, which are cut by the Sun's wire, and the Moon's, come to the East and West sides

sides of the horizon, the hour-hand points to the times of their rising and setting, on the day of the year which the wire then cuts that carries the Sun.

The wheel-work of this clock is as follows:

In the center, behind the middle of the Dial-plate, is a fixed pinion of 16 leaves, round which one wheel of 100 teeth, and another of 70, are carried, every 24 hours; the leaves of the pinion taking into the teeth of the wheels. On the axis of the wheel of 100 teeth is a pinion of 14 leaves, which turns a wheel of 69 teeth, on whose axis is a pinion of 7 leaves, turning a wheel of 83 teeth, which wheel is pinned to the back of the sydereal flat plate above-mentioned, which has the months and signs, &c. upon it; and any given point in the edge of this last wheel and plate, revolves from the meridian to the meridian again, in 23 hours, 56 min. 4 sec. 6 thirds (which makes a sydereal day) and from the Sun to the Sun again (which revolves in 24 hours),

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in 365 days, 5 hours, 48 minutes, 58 seconds, and 47 thirds, of mean solar time.

The above-mentioned wheel of 70 teeth (which is carried round the fixt pinion of 16 leaves every 24 hours) has a pinion of 8 leaves on its axis, which turns a wheel of 54 teeth; and to the axis of this wheel of 54 the Moon's wire is fixed, which carries the Moon round, from the meridian to the meridian again, in 24 hours*, 50 min. 25 seconds; round the ecliptic on the flat plate in 27 days, 7 hours, 43 minutes, and round from the Sun to the Sun again (or from change to change) in 29 days, 12 hours, 45 minutes.

II. About ten years ago, I made a wooden model of a clock for shewing the apparent diurnal motions of the

* It is generally believed that the Moon revolves from the meridian to the meridian again, in 24 h. 48 min. but that is a mistake: for if she did, there would be 30 complete days from change to change.

Sun and Stars, with the times of their rising and setting for every day of the year; and the days of the months all the year round, without any need of shifting by hand in the short months, as is always done in common clocks. I copied the Dial-plate of this model from a clock that Mr. Ellicott had made for the king of Spain: but although Mr. Ellicott shewed me the whole inside of the clock, I did not ask him what the numbers of teeth in the wheels of it were, although, I am convinced, he would have told me, if I had; nor do I, in the least, remember how many wheels there were in the uncommon or Astronomical part of it; and so I set about contriving wheels and numbers for performing the like motions.

The Dial-plate contains twice twelve hours, and within the circle of hours there is a large opening in the plate, a little elliptical: the edge of this opening serves for an horizon.

Below the Dial-plate, and seen through the large opening in it, is a
flat

flat plate on which the equator, ecliptic, and tropics, are drawn; and all the stars of the first, second, and third magnitudes are laid down, that are visible in the horizon (of Madrid in Mr. Ellicott's, and of London in mine) according to their right ascensions and declinations: the center of the plate being the North pole. The ecliptic is cut out into a narrow groove in the plate; and a small Sun slides in the groove by a pin, and is carried round by a wire fixt in the axis, which comes a little way out through the center of the plate. The edge of this plate is divided into the months and days of the year, and the Sun's wire shews the days of the months in these divisions. This star plate goes round in a sydereal day, making 366 revolutions in a year; in which time the Sun makes 365, and consequently shifts a division, or day of the month, every 24 hours.

A small wire is stretched from over the center of the sydereal plate to the
upper

upper XII on the fixed Dial-plate. This wire is for the meridian.

When the Sun, or any star, comes to the Eastern edge of the horizon, the hour index is at the time of rising of the Sun or Star, for the day of the year, pointed to by the wire, that carries the Sun : and when the Sun or Star comes to the Western edge of the horizon, the hour index is at the time of its setting. The Sun always comes to the meridian at the instant of the solar noon ; but every star comes sooner to the meridian every day, than it did on the day before, by 3 minutes, 55 seconds, 54 thirds of mean solar time, as it revolves from the meridian to the meridian again in 23 hours, 56 m. 4 s. 6 thirds.

When any star is on the meridian in the clock, the star which it represents is on the meridian in the heavens ; the time whereof is seen by the hour index on the Dial-plate. And, as the stars have their revolutions on the plate, one may look at the clock at any time, and see

see what stars are then above the horizon, what stars are then on the meridian, and what stars are then rising and setting.

My contrivance for shewing these motions and phenomena, in the model, consists of no more than two wheels and two pinions, as follows:

The wheels are of equal diameters, and so are the pinions; the numbers of teeth are 61 in one wheel, and 73 in the other. The pinions are both fixt on one axis, the one having 20 leaves and the other 24. The wheel of 73 teeth is fixed to the back of the sydereal wheel, and the axis of the wheel of 61 comes through the wheel of 73, and through the sydereal plate, and carries the wire round on which the Sun slides in the ecliptic groove, and also the hour hand on the Dial-plate.

The wheel of 61 teeth turns the pinion of 20, and the pinion of 24, (fixt on the same axis with that of 20) turns the wheel of 73.

Now, if the wheel of 61 teeth be turned round in 24 hours, to carry the

Sun and hour hand, the wheel of 73 teeth will be turned round in 23 hours, 56 minutes, 4 seconds, 6 thirds. And so, the sydereal plate will make just 366 revolutions, in the time that the Sun makes 365.

Mr. Ellicott had the prettiest, and most simple contrivance I ever saw, in his clock, for shewing the difference between equal and solar time (generally termed the equation of time) on all the different days of the year. He generously allowed me to copy that part into my model, and I have quite concealed it within one of my wheels, not to shew how it is done unless he publishes an account of it. The Sun, by that simple contrivance, even in my model, comes as much sooner or later to the meridian, than when it is Noon by a well-regulated clock, as the Sun in the heavens does, at all the different times of the year, excepting the four days on which the time of Noon shewn by the Sun and clock ought to coincide: and then there is no difference in

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the clock. And although the wheel-work is quite open to sight in the model which I now shew in my lectures, no person who sees it can guess how the unequal motion of the Sun, in the model, is performed.

III. In the year 1764, when I happened to be at Liverpool, I contrived a clock for Captain Hutchinson, who is Dock-master of the place, for shewing the age and phases of the Moon, and the time of High and Low water at Liverpool, every day of the year, with the state of the tides at any time of the day; by looking at the clock.

At the right and left lower corners of the Dial-plate, under the common circles for the hours and minutes, there are two small circular plates. On the plate at the left hand there are two circular spaces, the outermost of which is divided into twice twelve hours, with their halves and quarters: within which, the second circular space is divided into $29\frac{1}{2}$ equal parts for the days of the Moon's

Moon's age; each day standing under the time of the Moon's coming to the meridian on that day, in the circle of 24 hours. An axis comes through the center of this plate, and carries two indexes round it in 29 days, 12 h. 45 min. or from change to change of the moon: and these indexes are set as far asunder, as the time of High water at Liverpool differs from the time of the Moon's coming to the meridian. So that, by looking on this plate in the Morning, one may see at what time the Moon will be on the meridian, and at what time it will be High water at the place.

On the right hand plate, around its edge, all the different states of the tides are marked, from High to Low, and from Low to High; and within these appellations is a shaded ellipsis, the highest points of which represent High water, and the lowest parts Low water. An index goes round this plate in the time of the Moon's revolving from the meridian to the meridian again; and,

at all different times, points out the state of the tide, as it may be then High or Low, rising or falling.

In the arch of the Dial-plate above the hour of XII, a blue plate rises and falls as the Tides do at Liverpool: and, over this plate, in a painted sky, a globular ball, half black, half white, shews the phases of the Moon for every day of her age, throughout the year.

The wheel-work for shewing these appearances is as follows.

A wheel of 30 teeth is fixed on the axis of the twelve-hour hand, and turns round with it. This wheel turns a wheel of 60 teeth round in 24 hours, and on its axis is a wheel of 57 teeth, which turns round in the same time, and turns a wheel of 59 teeth round in 24 hours $50\frac{1}{2}$ minutes, on whose axis is the index on the right hand corner-plate, going round the plate in the time of the Moon's revolving from the meridian to the meridian again; and shewing the state of the Tide at any time, when the clock is
looked

looked at. On the axis of the same wheel of 59 teeth is fixed an elliptical plate, which raises and lets down the Tide-plate in the arch, twice in 24 hours $50\frac{1}{2}$ minutes; in which time there are two ebbings and flowings of the Tides.

The above-mentioned wheel of 57 teeth has a pinion of 16 leaves on its axis, turning a wheel of 70 teeth, on whose axis is a pinion of 8 leaves, turning a wheel of 40 teeth, which turns a wheel of 54 teeth round in 29 days, 12 hours, 45 minutes; and on the axis of the wheel of 54 teeth are the two indexes on the left hand corner-plate, for shewing the Moon's age on that plate, with the time of her southing, and of High water.

The wheel of 40 teeth here mentioned, might have been of any other number, and might have been left out, if the pinion of 8 leaves had taken into the wheel of 54 teeth: but then the index would have gone the wrong way round the dial-plate. So that the only
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use of the wheel of 40 is to be a leading wheel, for turning the index round the right way.

On the axis of the wheel of 54 teeth (which turns round in a lunation) is a small wheel of 20, turning a contrate wheel of the same number, on whose axis is the globular Moon (half black, half white) in the arch, turning round in a lunation, and shewing all her phases.

In these three clocks, I have only described the uncommon parts, which are connected with the common part of the movement known unto every Clock-maker.

An easy way of representing the apparent diurnal motions of the Sun and Moon in a clock.

Let a thick wheel of 57 teeth be turned round in 24 hours, and take into the teeth of two wheels of equal diameters, one of which has 57 teeth and the other 59; these wheels lying
5 close

close upon one another, and the axis of the one turning within the axis of the other. A wire fixt on the axis of the wheel of 57 will carry a Sun round in 24 hours; and a wire fixt on the axis of the wheel of 59 will carry a Moon round in 24 hours $50\frac{1}{2}$ minutes. If the Sun carries a plate round with him in 24 hours, and the limb of the plate be divided into $29\frac{1}{2}$ equal parts for the days of the Moon's age, the Moon will shew her age in the divisions of that plate; and may be made to turn round her axis, and shew her phases, by a wheel of any number of teeth, on her axis, and taking into the teeth of a contrate wheel of the same number, fixt on the axis of the wheel of 57 teeth, which carries the Sun.

An easy way of shewing the phases of the Moon, in a clock.

Let a wheel of 16 teeth be fixed on the axis of a wheel of 15, and the wheel of 16 turn a wheel of 63, on whose

whose axis let a ball, half black, half white, be fixed; and project half way out, through a round hole in the dial-plate.

Then, if the wheel of 15 teeth be always moved one tooth in 12 hours, the ball will be turned round in 29 days, 12 hours, 45 minutes, and shew all the various phases of the Moon.

An easy method of shewing the Sun's place in the Ecliptic every day of the year, in a clock; and his motion round the Ecliptic in a Solar year.

Let a pinion of 12 leaves be turned round once every ten hours, and this pinion take into a wheel of 67 teeth, on whose axis let there be a single threaded screw taking into a wheel of 157 teeth. This last wheel will turn round in 365 days, 5 h. 49 m. 50 sec. And an index on its axis will carry a Sun through the whole 360 degrees of an ecliptic, engraven on the dial-plate, in the same time: and may shew the

the days of the months on another circle within the ecliptic. This was the contrivance of Mr. *Arnsbaw* near *Manchester*, who communicated it to me.

How to shew the periodical revolutions of the Earth, and all the other planets, round the Sun, in a Clock; so as to agree nearly with the periodical revolutions of the planets about the Sun in the Heavens.

Let six hollow sockets, or arbors, be made to fit and turn within one another, and all of them to turn upon a fixt spindle, or axis; on the top of which let there be a ball to represent the Sun. Let the widest arbor be the shortest, and have an arm on its uppermost end to carry a ball representing Saturn, and a wheel of 206 teeth on its lowermost end.

Let the next sized arbor be so much longer than the above one, as to have a wheel (of 83 teeth) put upon it, below the wheel of 206; and an arm on

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the

the other end (above Saturn's) for carrying a ball to represent Jupiter.

Let the third socket be so much longer than the second, as to have a wheel on it (of 47 teeth) below the wheel of 206, and an arm on its other end, above Jupiter's, for carrying a ball to represent Mars.

Let the fourth arbor be so much longer than the third, as to have a wheel (of 40 teeth) on its lower end, and an arm on its upper end, above Mars's, for carrying a ball to represent the Earth.

Let the fifth arbor be so much longer than the fourth, as to have a wheel (of 32 teeth) on its lower end, below the wheel of 40, and an arm on its upper end, above the Earth's, for carrying a ball to represent the planet Venus.

Let the sixth (which is the smallest) arbor, be so much longer than the fifth, as to have a wheel (of 20 teeth) on its lower end, below the wheel of 32, and an arm on its upper end, above Venus's,

Venus's, for carrying a ball to represent the planet Mercury.

Saturn's arm must be the longest of all, because that planet is the furthest of all from the Sun: Jupiter's the next longest, Mars's the next, the Earth's the next, Venus's the next, and Mercury's the next, or shortest of all, because Mercury is the nearest of all the planets to the Sun.

The wheels must be fixed on their respective arbors, and diminish in their sizes from the highest numbers to the lowest; so that, when they are all put together, they may form somewhat of the appearance of a cone.

And, to give these wheels and planets their proper motions, they must be turned by six wheels (or rather four wheels and two pinions) all fixed on one solid axis, in a conical manner, inverted with respect to the other six wheels; so as the wheels and pinions on the solid axis may take into those on the arbors, and turn them.

The solid axis, with all its wheels and pinions, will turn round in the same time together, because the wheels and pinions are all fixed on the axis; and must be turned round once in a year by the clock-work; which may be easily done by such a method as Mr. *Arnshaw's*, already mentioned.

Then, if the uppermost and smallest pinion on the axis has 7 leaves, taking into Saturn's wheel of 206 teeth; Saturn will be carried round the Sun in 10748 days, 18 hours, 43 minutes: for, as 7 is to 206, so is 365.25 to 10748.78.

If the next pinion on the axis (which must be of a bigger size than the pinion of 7 above it) has 7 leaves, and takes into Jupiter's wheel of 83 teeth; Jupiter will be carried round the Sun in 4330 days, 19 hours, 40 minutes: for, as 7 is to 83, so is 365.25 to 4330.82.

If the wheel below this pinion on the axis has 25 teeth, and takes into Mars's wheel of 47; Mars will be carried round the Sun in 686 days, 16 hours,

hours, 5 minutes : for, as 25 is to 47, so is 365.25 to 686.67.

If the next bigger wheel on the axis, which turns round in 365.25 days, has 40 teeth, and takes into the Earth's wheel of 40 teeth; the Earth will be carried round the Sun in 365.25 days.

If the next bigger wheel has 52 teeth, and takes into Venus's wheel of 32 teeth; Venus will be carried round the Sun in 224 days, 18 hours, 29 minutes: for, as 52 is to 32, so is 365.25 to 224.77.

And lastly, if the largest wheel on the axis has 83 teeth, and takes into Mercury's wheel of 20; Mercury will be carried round the Sun in 88 days, 6 hours, 14 minutes : for, as 83 is to 20, so is 365.25 to 88.01.

I have seen a calculation of this sort in a printed book; but the numbers there are so faulty for Mars and Saturn, that I was obliged to alter them; Saturn's period being wrong by 51 days. How near these are to the truth will appear by comparing them with the annual periods in the following Table.

A Table

A Table shewing the times contained in the annual and diurnal revolutions of the Planets; with their relative and true distances from the Sun, the circumferences of their Orbits, the number of miles they move every hour therein, and their daily mean motions in degrees and parts of a degree. Their distances in miles from the Sun are here set down, as they were found to be by the Transit of Venus over the Sun, June 6, 1761.

The fix primary Planets.	Their annual Periods round the Sun.	Their diurnal Rotations.	Hourly motion of their Equators.	Their relative mean dist. fr. the Sun.	Their real distances from the Sun in English miles.	The circumferences of their Orbits in English miles.	The number of miles they move in each hour, in their Orbits.	Their daily mean motions in their Orbits.
d.	h.		Miles.	Parts.			Eng. Miles.	° ' "
Mercury	87 23	Unknown.	Unknown.	3871	36,841,468	231,574,940	109699.16	4 5 32
Venus	224 17	24 ^d . 8 ^h .	43	7233	68,891,486	433,032,198	80295.24	1 30 8
The Earth	365 6	1 ^d . 0 ^h .	1042	10000	95,173,000	598,230,286	68243.24	0 59 8
Mars	686 23	24 ^h . 40 ^m .	556	1237	145,014,148	911,517,502	55287.00	0 31 27
Jupiter	4332 12	9 ^h . 56 ^m .	25920	52009	494,990,976	3,111,371,849	29083.60	0 4 59
Saturn	10759 7	Unknown.	Unknown.	95400	907,956,130	5,707,152,817	22101.64	0 2 1

To assist the imagination in forming an idea of the vast distances of the planets from the Sun, let us suppose, that a body projected from the Sun, should continue to fly with the swiftness of a cannon ball, *viz.* 480 miles every hour; this body would reach the orbit of Mercury in 8 Julian years, 276 days; of Venus, in 16 years, 136 days; of the Earth in 22 years, 226 days; of Mars, in 34 years, 170 days; of Jupiter, in 117 years, 234 days; and the orbit of Saturn, in 215 years and 286 days.

If the reader should think this idea too extensive (notwithstanding its being a just one) he may contract it in the following manner, which takes in both the proportional bulks and distances of the Sun and Planets.

The Dome of St. Paul's is 145 feet in diameter. Suppose a globe of this size to represent the Sun: then, a globe of $9\frac{7}{10}$ inches will represent Mercury; one of $17\frac{9}{10}$ inches, Venus; one of 18 inches, the Earth; one of 5 inches diameter, the Moon (whose distance from

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the Earth is 240000 miles) one of 10 inches, Mars; one of 15 feet, Jupiter; and one of $11\frac{1}{2}$ feet, Saturn, with his ring four feet broad, and at the same distance from his body, all around.

In this proportion, suppose the Sun to be at St. Paul's; then Mercury might be at the Tower of London; Venus at St. James's Palace; the Earth at Marybone; Mars at Kensington; Jupiter at Hampton-Court, and Saturn at Cliefden: all moving round the Cupola of St. Paul's as their common center.

A PROBLEM.

I.

Suppose there are six hands on the Dial-plate of a clock, all going round the same way; and that the first or slowest hand, A, goes round in 24 hours; the next slowest hand, B, in 22 hours; the next, C, in 20 hours; the next, D, in 18; the next, E, in 16; and the last, or swiftest, F, in 14 Hours: and that they all set off together, from a conjunction, at any given point of the Dial-plate. Qu. In how many hours afterward will they all be in conjunction again, and how many revolutions will each hand have made in that time?

Let a, b, c, d, e, f , be the periodical times or revolutions of A, B, C, D, E, F , then, a will be 24 hours, b 22, c 20, d 18, e 16, and f 14.

I. The canon for finding the time that must elapse between the conjunctions

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junctions

junctions of A and B , is $\frac{a b}{a-b}$, and consequently all its multiples; *viz.* $\frac{2 a b}{a-b}$, $\frac{3 a b}{a-b}$, $\frac{4 a b}{a-b}$, &c. on to $\frac{m a b}{a-b}$; where m is any indefinite number of conjunctions.

For, if A and B are in conjunction at the end of the time $\frac{a b}{a-b}$, 'tis evident they will be in conjunction again when as much more time has elapsed; and so on to infinity.

2. The canon for finding the times between the conjunctions of B and C is $\frac{b c}{b-c}$ and all its multiples indefinitely; as $\frac{2 b c}{b-c}$, $\frac{3 b c}{b-c}$, &c. to $\frac{n b c}{b-c}$.

And therefore, when $\frac{a b}{a-b}$, or its multiple, is equal to $\frac{b c}{b-c}$ or its multiple, A , B , and C will then be in conjunction again. For, by the first expression, A and B will be in conjunction; and by the second, B and C will be so too. But the expressions being equal, the times must also be equal: that is, A , B , and C will be in conjunction again.

3. The canon for finding the times between the conjunctions of C and D

is

is $\frac{c d}{c-d}$, and all its multiples indefinitely, as above. And therefore, when any multiple of the conjunctions of A , B , and C , is equal to any multiple of the conjunctions of C and D , then A , B , C , and D , will be in conjunction again.

4. The canon for finding the times between the conjunctions of D and E is $\frac{d e}{d-e}$, and all its multiples; and therefore when any multiple of the conjunctions of A , B , C , and D , is equal to any multiple of the conjunctions of D and E , then, A , B , C , D , and E , will be in conjunction again.

5. The canon for finding the times between the conjunctions of E , and F , is $\frac{e f}{e-f}$, and all its multiples indefinitely; and therefore, when any multiple of the conjunctions of A , B , C , D , and E , is equal to any multiple of the conjunctions of E , and F , all the hands A , B , C , D , E , and F , will be again in conjunction.

The multiples must all be whole numbers, and the least that will do must

be taken, to find the times between the next succeeding conjunctions.

The times between the conjunctions of these six hands, taking them by two and two, are as follows ;

$\frac{a\ b}{a-b} = 264$; the number of hours in which *A* and *B* will come to their next conjunction, after their first setting out together.

$\frac{b\ c}{b-c} = 220$; the number of hours in which *B* and *C* will come to their next conjunction, after their first setting out together.

$\frac{c\ d}{c-d} = 180$; the number of hours in which *C* and *D* will come to their next conjunction, after their first setting out together.

$\frac{d\ e}{d-e} = 144$; the number of hours in which *D* and *E* will come to their next conjunction, after their first setting out together. And,

$\frac{e\ f}{e-f} = 112$; the number of hours in which *E* and *F* will come to their next conjunction,

conjunction, after their first setting out together.

In working for such multiples (of integer numbers) as will make the above expressions equal, in the least ratios of the times, I find they are as follows :

264 (or $\frac{a}{a-b}$) multiplied by 5, is equal to 220 (or $\frac{b}{b-c}$) multiplied by 6; equal to 1320 hours, for the time in which *A*, *B*, and *C*, will come to their next conjunction, after their first setting out together. And

1320 hours (the conjunction of *A*, *B*, and *C*) multiplied by 3, is equal to 180 hours (or $\frac{c}{c-d}$) multiplied by 22; equal to 3960 hours, for the time of the next conjunction of *A*, *B*, *C*, and *D*, after their first setting out together. And

3960 hours (the Conjunction of *A*, *B*, *C*, and *D*,) multiplied by 2, is equal to 144 hours multiplied by 55; (or $\frac{d}{d-e}$) equal to 7920 hours, for the time of the next conjunction of *A*, *B*, *C*,
D,

D, and *E*, after their first setting out together. And

7920 hours (the last mentioned conjunction) multiplied by 7, is equal to 112 hours (the conjunction of *E*, and *F*, or $\frac{ef}{e-f}$) multiplied by 495; equal to 55440 hours, the time in which all the six hands, *A*, *B*, *C*, *D*, *E*, and *F*, will be in conjunction again, after the instant of their first setting out together, from a conjunction at any given point of the dial-plate, and all moving round the same way, in the times above mentioned.

Now, as it will require 55440 hours (or 2310 days) to bring all these hands together again, after their first setting out together; divide 55440 hours by the number of hours in which each hand goes round, and the quotients will shew that *A* has made 2310 revolutions, *B* 2520, *C* 2772, *D* 3080, *E* 3465, and *F* 3960. And, at the end of so many more revolutions of each hand, they will all be in conjunction again; and so on continually.

II.

The periodical times of the six primary planets being given, and supposing them to have been all at once in a line of conjunction with the Sun; to find how much time would elapse before they were all in a line of conjunction with the Sun again. This Problem I had from Mr. Waring (now Professor of Mathematics in the University of Cambridge) in the year 1755.

Let a, b, c, d, e, f , be respectively equal to the periodical times or revolutions of the six planets about the Sun; a being the longest, or Saturn's period; b the next longest, or Jupiter's; c the next, or Mars's; d the next, or the Earth's; e the next, or Venus's; and f the shortest period of all, which is Mercury's: and let $p, qp, rqp, srqp$, &c. be equal to the difference or time between the succeeding conjunctions of any two, three, four, &c. of them.

'Tis evident that q, r, s , (the multipliers) must be whole numbers, because the numbers of conjunctions are so.

The time between the conjunctions of the first two is $\frac{a b}{a-b}, = p$; that of the first three is $\frac{n \times a c}{a-c}, = qp$ (where n is any number assumed, to make q a whole number) or, which is the same, $\frac{n a c}{a-c \times p} = q$; $\frac{a c}{a-c \times p}$ being reduced to its lowest denominator, q will be equal to the numerator of that fraction. In the same manner, r will be equal to the numerator of the fraction $\frac{a d}{a-d \times qp}$ reduced to its lowest denominator; s will be equal to the numerator of the fraction $\frac{a e}{a-e \times rqp}$, reduced to its lowest denominator; and so on, from the slowest to the quickest revolving bodies in the system: by which means, the times of all their conjunctions may be found.

This

This Problem may be solved by a different method, as follows; for which I am obliged to my generous friend Mr. John Ford, Surgeon in Bristol.

Let A, B, C, D, E, F , stand for the six planets, beginning with Saturn and ending with Mercury; and a, b, c, d, e, f , be the times of their periodical revolutions respectively. Then, by a known rule, the synodical period, or conjunction, of A and B , will be the time $\frac{ab}{a-b}$; and that of B and C will be $\frac{bc}{b-c}$; that of C and D will be $\frac{cd}{c-d}$; that of D and E will be $\frac{de}{d-e}$; and that of E and F will be $\frac{ef}{e-f}$.

Now it is obvious, that A and B can never be in conjunction but in the time $\frac{ab}{a-b}$, or some multiple of it; neither can B and C be in conjunction but in the time $\frac{bc}{b-c}$, or some multiple of that time. A, B , and C , will therefore be in conjunction when $\frac{mab}{a-b}$ is equal to $\frac{nbc}{b-c}$, where m and n represent two integer numbers, prime to each other; which being re-

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spectively

spectively multiplied into $\frac{a\ b}{a-b}$ and $\frac{b\ c}{b-c}$ shall make the two products equal. And these two numbers are easily discovered; for, as by supposition, $\frac{m\ a\ b}{a-b}$ is equal to $\frac{n\ b\ c}{b-c}$, therefore, $m.n :: \frac{b\ c}{b-c} : \frac{a\ b}{a-b}$. Reduce therefore $\frac{b\ c}{b-c}$ and $\frac{a\ b}{a-b}$ into integers of the least dimensions, which shall have the same proportion to each other as these numbers have; and you will have the multipliers m and n , and consequently the synodical period or conjunction of A , B , and C ; which we shall call R . In the same manner may the synodical period of C , D , and E , be investigated, which call S : then find two prime numbers r and s in their lowest dimensions, which shall have the same proportion to each other as the times R and S ; then will rS , or its equal Sr , give the synodical period, or conjunction of the five planets, A , B , C , D , and E , which characterize by T . Find lastly, the synodical period of E and F , by the rule $\frac{e\ f}{e-f}$, which denote by X ; and the least integer numbers

bers t , x , in the same proportion to each other as T and X being found, tX or xT will be the synodical period, or conjunction, of the six primary planets, A, B, C, D, E, F ; or the time that must elapse between any conjunction of them all, and the next succeeding conjunction. Which time, being divided by the time of the periodical revolution of each planet, will shew how many revolutions each planet has then made.

There are several ways of finding the above-mentioned prime integer numbers or multipliers; but the following is very convenient and easy.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two of the fractions. Multiply the denominator of the first into the numerator of the second, and *vice versa*; then strike out both the denominators, by which process the above fractions become ad and bc ; which numbers are in the same proportion as the fractions; and, if they are prime to each other, are the numbers required. But if they are not

U 2 prime,

prime, divide them by their greatest common divisor, in order to reduce them to their lowest denomination.

The reason why these numbers must be prime integers is plain: for, if they were not so, we should not have the synodic period required, but some multiple of it: and if they were not integers, we should not have exact multiples of the lower synodic periods from which we deduce the higher.

To facilitate calculations which may be made on these principles, I shall subjoin the following Table, which shews the annual periods of the primary planets, reduced to hours; and their synodical periods, taken two by two progressively. But although the synodical periods of the planets, taken two by two, is so short, it must not be imagined that the synodical periods of three planets must be proportionably so too. The synodic period of the Earth and Venus (by the Table) is 1 year, 218 days, 17 hours; and that of Venus and Mercury is 144 days, 12 hours; but

but the synodical period of these three planets is upwards of 5500 years.

If the periods of three planets be so incommensurate, how much more so must be the periods of the fix revolving primaries of our system? Indeed we here cannot but see and admire the wisdom and providence of the SUPREME BEING! For, had the times of the annual revolutions of the several planets been more commensurate, the present arrangement of our system would doubtless have been greatly disturbed by the conspiring attraction of the fix bodies, when they happened to be in conjunction; an arrangement, which, from the goodness of the Almighty, we must conclude to be, in its present state, the best adapted to answer the purposes for which the system was created.

Names of the Planets.	—	Their periodical revolutions reduced to hours.	
Saturn	—	258223 ^h	= <i>a</i>
Jupiter	—	103980	= <i>b</i>
Mars	—	16487	= <i>c</i>
Earth	—	8766	= <i>d</i>
Venus	—	5393	= <i>e</i>
Mercury	—	2111	= <i>f</i>

Their

Their synodical periods, or conjunctions with each other.

	Y.	D.	H.	Hours.
Saturn and Jupiter } Jupiter and Mars }	19	313	10	$= 174076 = \frac{a}{a-b}$
Jupiter and Mars }	2	85	21	$= 19593 = \frac{b}{b-c}$
Mars and the Earth }	2	49	10	$= 18718 = \frac{c}{c-d}$
Earth and Venus }	1	218	17	$= 14015 = \frac{d}{d-e}$
Venus and Mercury }	0	144	12	$= 3468 = \frac{e}{e-f}$

To illustrate the use of this Table, let it be required to find the synodical period or conjunction of the Earth, Venus, and Mercury.

That of the Earth and Venus 14015 hours, $= \frac{de}{d-e}$; and that of Venus and Mercury is 3468 hours, $= \frac{ef}{e-f}$.

Therefore, from what has been already laid down (see page 145) the synodical period of the three planets will be when $m \times 14015$ is equal to $n \times 3468$; or when $m:n::3468:14015$; m and n being the least integer numbers in the proportion of 3468 to 14015. But these numbers being integers, and

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in their lowest terms already, they require no reduction. Therefore, 3468×14015 give the synodical period of the three planets, $= 48604020$ hours; $= 5544$ years, 221 days, 12 hours. The reader may proceed to find out the synodic periods or conjunctions of the rest, according to the foregoing rules.

The following problem is of the same nature with this; but, as it is more familiar and obvious, it may better serve to confirm the truth of the method we have used to investigate the synodical period of bodies revolving the same way, but in different times, about the same common center.

III.

Suppose the hour, minute, and second hands of a Clock to be in conjunction at the hour of XII. It is required to find when they will be in conjunction again?

Here we have the periodical revolution of the hour-hand $= 720$ minutes, $= a$; the periodical revolution of the
1
minute-

minute-hand = 60 minutes, = b ; and
that of the second-hand = 1 minute,
 = c : from whence we collect $\frac{a b}{a-b}$,
 $= \frac{43200}{660}$ min. $= \frac{7200}{110} = \frac{720}{11}$ min. for the syno-
 dical period or conjunction of the hour
 and minute hands; and $\frac{b c}{b-c} = \frac{60}{59}$ for the
 synodical period of the minute and se-
 cond hands. Then, to find the syno-
 dical period of all the three hands, we
 must (as in the above Problem) suppose
 $\frac{m \times 720}{11} = \frac{n \times 60}{59}$; from whence we have
 $m : n :: \frac{60}{59} : \frac{720}{11}$. Now, the least integer
 numbers, represented by m and n , in
 the proportion of $\frac{60}{59}$ to $\frac{720}{11}$ are 11 and
 708. Therefore $11 : 708 :: \frac{60}{59} : \frac{720}{11}$; and
 consequently $\frac{11 \times 720}{11} (= 708 \times \frac{60}{59}) =$ the
 synodical period of the three hands of
 the Clock; = 720 minutes, or just 12
 hours.

The periodical revolutions of the Sun
 and Moon round the Ecliptic, and
 their synodical periods or conjunctions
 with each other, may be familiarly re-
 presented by the motions of the hour
 and

and minute hands of a watch, round its Dial-plate. For, the Dial-plate is divided into 12 hours, as the Ecliptic is divided into 12 signs; the hour-hand goes round in 12 hours, as the Sun does in 12 months, and the minute hand goes round in 1 hour, as the Moon does in (somewhat less than) a month. And, as the Moon is never in conjunction with the Sun in that point of the Ecliptic where she was at the last conjunction before, so the minute-hand never is in conjunction with the hour-hand at that point of the Dial-plate where it was at the last preceding conjunction. So that, the 12 hours on the Dial-plate may represent the 12 signs of the Ecliptic; the hour-hand the Sun, and the minute-hand the Moon: only, the motion of the minute-hand is too slow for the Moon in proportion to *that* of the hour-hand compared with the motion of the Sun. For, in the time of the Sun's going round the Ecliptic, which is 12 calendar months, there are 12.36 conjunctions of the

Sun and Moon; but in the time the hour-hand goes round the Dial-plate, the minute-hand is only 11 times conjoined with it.

These hands are always in conjunction at XII o'clock. The first column of the Table shews the number of their conjunctions in 12 hours, and the collateral lines shew in how many hours, minutes, &c. after XII, they come to their succeeding conjunctions marked in the first column; the time between any conjunction and the next being 1 hour, 5 $\frac{5}{11}$ minutes.

Conj.	Hou.	m.	II	III	IV	V	VI	VII	VIII	IX	X
1	I	5	27	16	21	49	5	27	16	21	49 $\frac{1}{11}$
2	II	10	54	32	43	38	10	54	32	43	38 $\frac{2}{11}$
3	III	16	21	49	5	27	16	21	49	5	27 $\frac{3}{11}$
4	IIII	21	49	5	27	16	21	49	5	27	16 $\frac{4}{11}$
5	V	27	16	21	49	5	27	16	21	49	5 $\frac{5}{11}$
6	VI	32	43	38	10	54	32	43	38	10	54 $\frac{6}{11}$
7	VII	38	10	54	32	43	38	10	54	32	43 $\frac{7}{11}$
8	VIII	43	38	10	54	32	43	38	10	54	32 $\frac{8}{11}$
9	IX	49	5	27	16	21	49	5	27	16	21 $\frac{9}{11}$
10	X	54	32	43	38	10	54	32	43	38	10 $\frac{10}{11}$
11	XII	0	0	0	0	0	0	0	0	0	0

If the above process was carried on to infinity, in the horizontal lines, the numbers would circulate at every sixth column.

To

*To represent the motions of Jupiter's four
Satellites round Jupiter, in a clock;
and shew the times of their Eclipses in
Jupiter's shadow.*

On four hollow arbors, let there be four bent wires of different lengths, to carry the Satellites round Jupiter, as the arbors are turned round within one another; and let Jupiter be fixed on the top of a solid axis or spindle, on which all the arbors are turned round; the wires being so bent, as that the Satellites, on their tops, may be of the same height with Jupiter's ball. The diameters of the Satellites should not be above a sixth or seventh part of the diameter of Jupiter; and, to be at their proper distances from him, the distance of the nearest Satellite should be $5\frac{2}{3}$ semidiameters of Jupiter from his center; the second Satellite, 9 semidiameters of Jupiter distant from his center; the third $14\frac{1}{3}$ semidiameters; and the fourth, $25\frac{1}{3}$ of his semidiameters from his center.

X 2

Let

Let four wheels of different sizes, and different numbers of teeth, be fixed upon the lower end of the abovementioned arbors, in a conical manner, as described in the former machine (pag. 129); the wheel on the smallest arbor, that carries the first Satellite, having 22 teeth; the wheel on the next arbor, that carries the second Satellite, 33 teeth; the next bigger wheel on the arbor that carries the third Satellite, 43 teeth; and the largest wheel of all, on the arbor that carries the fourth (or outermost Satellite), 67: the biggest wheel being the uppermost, and the smallest the lowermost.

These four wheels must be turned by other four, all fixt on a solid axis, in an inverted conical manner, with respect to the former wheels on the hollow arbors; and then, all the four on the solid axis will be turned round in one and the same time.

The smallest wheel (or uppermost one) on this axis, must have 28 teeth;
and

and turn the wheel of 67 teeth, which carries the fourth Satellite.

The next wheel on the axis must have 42 teeth; and turn the wheel of 43 teeth, which carries the third Satellite.

The next bigger wheel below, on the axis, must have 65 teeth; and turn the wheel of 33 teeth, which carries the second Satellite. And,

The lowermost, and biggest wheel on the axis, must have 87 teeth; and turn the wheel of 22, which carries the first Satellite. Then,

If the clock turns the solid axis with all its wheels round in 7 days, the first Satellite will be carried round Jupiter in 1 day, 18 hours, 28 minutes, 57 seconds; the second Satellite, in 3 days, 13 hours, 17 minutes, 46 seconds; the third, in 7 days, 3 hours, 59 minutes, 54 seconds; and the fourth Satellite in 16 days, 18 hours, 0 minutes, 0 seconds; which agrees so nearly with their revolutions in the heavens, as not to differ sensibly, in a long time, from them.

And then, if a piece of black wood be turned, a little conical in its shape, having its thickest end as broad as the diameter of Jupiter is long, and be made hollow to fix on the back of Jupiter, and have notches cut in it for the Satellites to pass through: it will represent Jupiter's shadow; and when the Satellites are in the notches, it will shew them to be eclipsed.

The times of the immersions of the Satellites of Jupiter into his shadow, or of their emersions from it, may be had from *White's Ephemeris* every year; and if the Satellites are once put just entering into the notches for the immersions, or just leaving it for the emersions, at the proper times by the clock; they will keep right to the times thereof for more than a year afterward, without needing any new adjustment. And, in order that they may be so set, without affecting the wheels that move them, their wires should be fixed into round collars, which go moderately tight on the tops of the four hollow arbors, so
as

as they may be carried about Jupiter by the tightness of the collars; and yet at any time may be moved, and set right by hand.

All the numbers of teeth in the wheels are here copied from Mr. *Romer's* Satellite instrument, except those for the second Satellite; where Mr. *Romer* has a wheel of 63 teeth, turning a wheel of 32 : instead of which, I make a wheel of 65 turn a wheel of 33, which comes much nearer the truth.

About 16 years ago, I made one of these instruments, to be turned with a winch by hand. It had a Dial-plate divided into the months and days of the year, within which was a circle divided into twice twelve hours. On this Plate there were two indexes, one of which was moved round, over all the 365 days of the annual circle, in 365 turns of the winch : and the other index was moved round, over all the 24 hours, in one turn of the winch ; by which means I could, in a very short time, shew at what times of the
days

days the Satellites would be eclipsed, throughout the whole year. And, after having the above numbers for the motions of the Satellites, any Clock-maker may easily construct a machine of this sort ; by which, the times of the Immersions or Emerfions of the Satellites may be known before-hand, in order to be prepared for observing them in the heavens.

How

How to construct an Orrery for shewing the annual revolutions of Mercury, Venus, and the Earth, round the Sun, in their proper periodical times; the Moon's motion round the Earth, and round her own axis, with all her different Phases: the motions of the Sun, Venus, and the Earth, round their respective axes; the vicissitudes of Seasons; the retrograde motion of the nodes of the Moon's Orbit; with the times of all the New and Full Moons, and of all the Solar and Lunar Eclipses.

Let a wheel of 6.12 inches, having 74 teeth, be fixed on the axis of the handle or winch, and turn a wheel of 1.80 inches diameter, having 32 teeth, which turns a wheel of 73 teeth, whose diameter is 6.11 inches; and, on the axis of this last wheel let there be one of 32 teeth, of 1.80 inches diameter, turning a wheel of 160 teeth, whose diameter is 8.97 inches; and that wheel to turn two wheels of 32 teeth
Y each,

each, and diameter 1.80 inches. One of these last wheels of 32 to have a small wheel of 16 teeth on the top of its axis, turning another of the same number and size, and that one to turn such another, on the top of whose axis (inclining $23\frac{1}{2}$ degrees) is the Earth, which turns round in the same time as the winch; each turn answering to 24 hours. The Earth is covered half over with a black cap, and turns freely round within the cap, whose edge represents the boundary of light and darkness, and shews the times of the [apparent] rising and setting of the Sun, as the different places of the Earth emerge from below it, or go in under it.

The other wheel of 32 teeth (turned by the foresaid wheel of 160) has an index on its axis, which goes round a Dial-plate of 24 hours, in the time the Earth turns round its axis. The same wheel of 32 turns one of 64 teeth, whose diameter is 3.60 inches, and turns a wheel of 30 teeth, 1.6 inches diameter;

diameter ; and on the axis of this wheel is a single threaded screw, turning a wheel of 63 teeth, whose diameter is 3 inches, and turns a wheel of 24 teeth, 1.23 inches diameter, which turns a wheel of 63 teeth, 3 inches diameter ; which last wheel carries the Moon round her orbit in 27 days, 7 hours, 43 minutes, and from change to change, in 29 days, 12 hours, 45 minutes. The first wheel of 63 teeth has an index on the top of its axis, which goes round a circle divided into $29\frac{1}{2}$ equal parts in the time of a Lunation ; and shews the Moon's age every day.

A small wheel of 20 teeth is fixed on a socket, among the other work, below the Earth ; and by the bar that carries the Moon, a wheel hanging on the bar, of 20 teeth, turns another of the same number and size, on whose hollow axis is the Moon's black cap, which always faces the Sun, and shews the Moon's phases, as she turns round her axis, which is within the hollow axis of her cap.

On the axis of the winch is a pinion of 8 leaves, turning a wheel of 25 teeth, which turns another of the same number, on whose axis is a pinion of 7 leaves, $\frac{62}{100}$ parts of an inch in diameter, which turns a wheel of 69 teeth, whose diameter is 4.12 inches, and has a pinion of 7 leaves on its axis, turning a wheel of 83 teeth, which is fixed to a frame that contains several of the above-mentioned wheels within it, and carries the Earth round the Sun in 365 days, 5 hours, 48 minutes, 57 seconds. The diameter of the wheel of 83 teeth is 6.12 inches,

On the axis of the last mentioned wheel of 69 teeth, is a pinion of 10 leaves, turning a wheel of 73 teeth, whose diameter is 5.82 inches, and is fixed to a frame in which are several other wheels (to be described by and by) and carries Venus round the Sun in 225 days, 17 hours.

On the axis of the foresaid wheel of 69 teeth is a wheel of 78, whose diameter is 3.68 inches, and turns a wheel
of

of 64 teeth, whose diameter is 2.3 inches, and on the top of whose axis the Sun is placed ; the axis inclining $7\frac{1}{2}$ degrees, and the Sun turning round by it in 25 days, 6 hours.

In the center of the machine, below the Sun, there are three wheels fixed on the stem, round which the whole work moves ; the stem itself being fixed into the bottom of the box which contains the work. The lowermost of these three wheels is 2.95 inches in diameter, and contains 50 teeth, which take into the teeth of another wheel of the same number and size, and this last wheel takes into the teeth of another of the same number and size, for keeping the parallelism of the Earth's axis in its whole course round the Sun ; on which parallelism, the whole variety of the Seasons depend.

On the axis of the middlemost of these three wheels of 50 teeth is a wheel of 59, (a little bigger than the wheel of 50) which takes into a wheel of 56 teeth (of a somewhat smaller size)

and this wheel of 56 moves the Nodes of the Moon's orbit backward, through all the signs and degrees of the Ecliptic in $18\frac{2}{3}$ years.

Above the fixed wheel on the middle stem, of 50 teeth, is a fixed wheel of 74, whose diameter is 6.12 inches, and takes into a pinion of 8 leaves, on the top of whose axis is a small wheel of 16 teeth, turning another of the same number and size, and that turning another of the same number and size also, on whose axis, inclining 75 degrees, Venus is turned round in 24 days, 8 hours; which is her diurnal period, according to *Bianchini's* observations.

Above the said wheel of 74 teeth, and fixed on the same stem, is a wheel of 28, whose diameter is 1.74 inches; this wheel takes into the teeth of another of the same number and size, which takes into a third of the same number and size also; and this third wheel keeps the parallelism of Venus's axis throughout her whole annual period round the Sun.

On

On the axis of the middlemost of these three wheels of 28 teeth is another of the same number and size, which turns a wheel of 18 teeth, whose diameter is 1.12 inches, and which turns another of the same number and size, which carries Mercury round the Sun in 87 days, 23 hours.

Any person who is not accustomed to the making of Orreries may perhaps be apt to think, that all the abovementioned motions might be performed by fewer wheels; and an expert Clock-maker, by computing the periodical times of the planets revolutions from the numbers of teeth in these wheels, might pronounce them to be very inaccurate. But it ought to be considered, that there is a very great difference between the rotations of wheels which always keep in the same places, and of those which do not only turn round, but are also carried round others, continually changing their places and positions. As I wanted an Orrery more exact in the annual periods of the planets,

nets, and motion of the Moon round her orbit, than any one I have yet seen; the common Orreries being more adapted for reading public lectures upon; where it is sufficient to shew and explain the general phenomena; the makers generally content themselves with having such numbers as will carry the Earth round the Sun in 365 of its diurnal rotations, the Moon round from change to change in $29\frac{1}{2}$ days, and her nodes round the Ecliptic in 19 years; I have taken the pains to calculate the abovementioned numbers, which are far more exact; and got a good workman to make an Orrery under my inspection, in which the diameters of the wheels, and their numbers of teeth are exactly described; and which I now give freely to those who choose to work by them.

Another

Another Orrery.

About twelve years ago, I made a large wooden Orrery, for shewing only the motions of the Earth and Moon, with the retrograde motions of her nodes, and the phenomena arising from all their motions. The Earth had not its diurnal and annual motions carried on by means of a winch, but by hand; and as the Earth was moved round the Sun, the Moon was carried round the Earth in her orbit, and her nodes had their retrograde motions. As there is something very particular and simple in the construction of this machine; and as the Moon's motion in it will not vary above one degree from the truth in 304 years; and as it answers as well in Leap years as in common years; and has only seven wheels and one pinion in it; I shall here mention its use, but must beg to be excused from describing the position of its wheels, and their numbers of teeth, because I intend to instruct my

Z son,

son if both he and I live till the proper time, how to make it for his own benefit. Besides, it would be very difficult to make it intelligible by a description, without seeing it ; especially as some of the wheels are not only divided into very uncommon numbers of teeth, but also that, in some of the wheels, equal numbers of teeth are contained in unequal spaces ; for shewing the inequality of the Earth's annual motion round the Sun, and of the Moon's motion round her orbit. It shews the following matters very readily.

The lengths of days and nights at all places of the Earth, and at all times of the year ; with all the vicissitudes of Seasons. The Sun's place in the Ecliptic on any given day of the year, and time of the day ; with his Declination, Altitude, and Azimuth at any time ; also his Amplitude, and the time of his rising and setting. The time of the day, by the observed Altitude or Azimuth of the Sun. The variation of the compass, in any place, whose latitude is

known by a single obſervation of the Sun's Altitude, taken at any time, either in the Forenoon or Afternoon. The Moon's periodical and ſynodical revolution, with her rotation on her axis, and different phaſes. The retrograde motion of her Nodes, and direct motion of her Apogee. Her mean Anomaly and elliptic Equation, by which her true place in her Orbit is very nearly found at any time. Her Latitude, Declination, Altitude, and Azimuth, at any time when ſhe is above the horizon. Her Amplitude, and the time of her riſing and ſetting, however affected by her Latitude. The times of all the New and Full Moons, and of all the Solar and Lunar Eclipses, within the limits of 6000 years before or after the Chriſtian Æra; with an eaſy method of rectifying the Machine, in leſs than two minutes of time, for the beginning of any given year within theſe limits: and when it is once rectified, it will keep right for 304 years either backwards or forwards; at the end of which

Z 2

time,

time, the Moon must be set one degree forward in her orbit. The small difference in the time of the Moon's rising, in Harvest, throughout the week in which she is Full; and the great difference in the time of her setting during that week. The Recession of the Equinoctial points, in the Ecliptic. The Phenomena of the Tides, and the causes of many apparent irregularities in their heights, and times of ebbing and flowing.

The Mechanical Paradox.

This is a small kind of Orrery, which I contrived and made about fifteen years ago. It has only five wheels, and shews the Seasons, the retrograde motion of the Moon's Nodes, and the mean times of Eclipses of the Sun and Moon. I gave it the above name, because there is one wheel in it as thick as three of the others; and *that* wheel takes fairly and equally deep into the teeth of these three other wheels (which are quite independent

dent of, and unconnected with, each other); and yet, the thick wheel affects the three wheels in such a manner, and at the same time, as to turn the uppermost of them forward, the middlemost backward, and the lowermost no way at all. For a Copper-plate of this machine, with a printed description in which the paradox is solved, I refer the reader to a Shilling Pamphlet, sold at Mr. Cadell's Shop, opposite Catherine Street in the Strand,

A short account of the Silk Mills at Derby.

In these Mills are 26586 wheels, and 97746 movements, continually working except on Sundays. This grand machine is disposed in four stories of large rooms above one another; and the whole is actuated by one great Water-wheel, which goes round three times in a minute. In each time of its going round, 73728 yards of Silk are twisted: so that, in 24 hours, 318504960 yards are

are twisted. The water-wheel is kept constantly going ; but on Sundays it is disengaged from all the rest of the work. Any part of these movements may be stopt without the least prejudice or interruption to the rest.

Wond'rous Machine ! Thy curious Fabric shews
 How far the power of human wisdom goes !
 Where many thousand movements all attend
 Upon a WHEEL, and on THAT Cause depend.
 Sceptic, advance ! propose *thy* scheme of wit,
 That faith to reason always must submit.
 Whence learn'd these movements to obey command ?
 Who taught them how to roll, and when to stand ?
 Was it by chance this curious fabric came ?
 Or did some thought precede, and rule the Frame ?
 Worthy the Mortal, on whose Soul, confess,
 His GREAT CREATOR's Image stands impress !
 Now turn from Earth to Heaven thy doubting eyes,
 And read th' amazing Glories of the Skies !
 Worlds without number roll in different Spheres,
 Keep to their Seasons and complete their years.
 Five thousand circuits, made with equal force,
 The Earth has finish'd by its annual Course.
 The Sun dispenses beams of genial Light,
 And lends his rays to cheer the gloomy night.
 STUPENDOUS POWER and THOUGHT ! Enquire no
 more :
 Own the FIRST MOVER ; and, convinc'd, ADORE.

Rules

Rules for finding the corresponding years of the Julian Period with the years of the world, and years before and since the birth of CHRIST; supposing (with Mr. Bedford, in his Scripture Chronology) that the Creation of the World was in the 706th year of the Julian Period; and that the birth of CHRIST was (according to the vulgar Æra thereof) in the 4713th year of the Julian Period.

From any given year of the Julian period subtract 706, and the remainder will be the years of the world's age.

If the number of the given year of the Julian period be less than 4713, subtract it from 4713; and the remainder will be the number of years before the year of Christ's birth.

If the given year of the Julian period is greater than 3967, subtract 3967 from it; and the remainder will be the number of years after the famous Æra of Nabonassar.

Subtract

Subtra $\text{\textcircled{c}}$ t 1 from any given year of the Julian period, and divide the remainder by 4; if nothing remains, the given year is a Leap year : but if 1, 2, or 3 remains, it is the first, second, or third year after Leap year, in the Old Stile.

If any year before the year of Christ's birth be given, subtra $\text{\textcircled{c}}$ t its number from 4713, and the remainder will be the year of the Julian period. And if you subtra $\text{\textcircled{c}}$ t the said given year from 4007, the remainder will be the years of the world's age.

If any year after the year of Christ's birth be given, add 4713 to it, and the sum will be the year of the Julian period; or if you add 4007 to it, the sum will be the years of the world's age.

If any year of the world's age is given, add 706 to it, and the sum will be the year of the Julian period. If the given year of the world be less than 4007, subtra $\text{\textcircled{c}}$ t it from 4007; and the remainder will be the number
of

of years before the year of Christ's birth. But, if the given year of the world be more than 4007, subtract 4007 from it; and the remainder will be the number of years after the year of Christ's birth.

A Table of remarkable Eras and Events.

	Julian Period.	World's Age.	Before Christ.
1. The Creation of the World	706	0	4007
2. The Flood — — —	2362	1656	2351
3. The Assyrian monarchy founded by Nimrod — — —	2537	1831	2176
4. The birth of Abraham — —	2714	2008	1999
5. The destruction of Sodom and Gomorrhah — — —	2816	2110	1897
6. The kingdom of Athens founded by Cecrops — — —	3157	2451	1556
7. Moses receives the ten command- ments from God — — —	3222	2516	1491
8. The Israelites enter Canaan	3262	2556	1451
9. The destruction of Troy — —	3529	2823	1184
10. The beginning of king David's reign — — —	3650	2944	1063
11. The founding of Solomon's Temple — — —	3701	2995	1012
12. The Argonautic Expedition	3776	3070	937
13. Lycurgus formed his excellent Laws — — —	3829	3103	884
14. Arbaces, first king of the Medes	3838	3132	875
15. Mandaucus, the second —	3865	3159	848
16. Sotarmus, the third — —	3915	3029	798
17. The beginning of the Greek Olympiads — — —	3938	3232	775
18. Artica, the fourth king of the Medes — — —	3945	3239	768
A a			19. The

	Julian Period.	World's Age.	Before Christ.
19. The <i>Catonian Epoch</i> of the building of <i>Rome</i> — — —	3961	3255	752
20. The <i>Æra</i> of <i>Nabonassar</i> — —	3967	3261	746
21. The destruction of <i>Samaria</i> by <i>Salmaneser</i> — — —	3992	3286	721
22. The first Eclipse of the Moon on record — — —	3993	3287	720
23. <i>Cadisea</i> , the fifth king of the <i>Medes</i> — — —	3996	3290	717
24. <i>Phraortes</i> , the sixth — — —	4058	3352	655
25. <i>Cyaxares</i> , the seventh — — —	4080	3374	633
26. The first <i>Babylonish</i> captivity by <i>Nebuchadnezzar</i> — — —	4107	3401	606
27. The long war ended between the <i>Medes</i> and <i>Lydians</i> — —	4111	3405	602
28. The second <i>Babylonish</i> captivity, and birth of <i>Cyrus</i> — — —	4114	3408	599
29. The destruction of <i>Solomon's</i> Temple — — —	4125	3419	588
30. <i>Nebuchadnezzar</i> struck with madness — — —	4141	3438	569
31. <i>Daniel's</i> vision of the four monarchies — — —	4158	3452	555
32. <i>Cyrus</i> begins to reign — — —	4177	3471	536
33. The battle of <i>Marathon</i> — —	4223	3517	490
34. <i>Artaxerxes Longimanus</i> begins to reign — — —	4249	3543	464
35. The beginning of <i>Daniel's</i> seventy weeks of years — —	4256	3550	457
36. The beginning of the <i>Peloponnesian</i> war — — —	4282	3576	431
37. <i>Alexander's</i> victory at <i>Arbela</i> — —	4383	3677	330
38. His death — — —	4390	3684	323
39. The captivity of 100,000 Jews by king <i>Ptolemy</i> — — —	4393	3687	320
40. The <i>Colossus</i> of <i>Rhodes</i> thrown down by an earthquake — —	4491	3785	222
41. <i>Antiochus</i> defeated by <i>Ptolemy Philopater</i> — — —	4496	3700	217
42. The famous <i>ARCHIMEDES</i> murdered at <i>Syracuse</i> — — —	4506	3800	207
43. <i>Jason</i> butchered the inhabitants of <i>Jerusalem</i> — — —	4543	3837	170
44. <i>Corinth</i> taken and plundered by Consul <i>Mummius</i> — — —	4567	3861	146

	Julian Period.	World's Age.	Before Christ.
45. <i>Julius Cæsar</i> invades Britain	4659	3953	54
46. He corrects the calendar	4667	3961	46
47. Is killed in the Senate-house	4671	3965	42
48. <i>Herod</i> made king of <i>Judea</i>	4673	3967	40
49. The battle at <i>Actium</i> —	4683	3977	30
50. <i>Agrippa</i> builds the <i>Pantheon</i> at <i>Rome</i> — — —	4668	3982	25
51. The true <i>Æra</i> of CHRIST's birth — — —	4709	4003	4
52. The death of <i>Herod</i> — —	4710	4004	3
			Since Christ.
53. The <i>Dionysian</i> , or vulgar <i>Æra</i> of CHRIST's birth — — —	4713	4007	0
54. The true year of his Cruci- fixion — — —	4746	4040	33
55. The destruction of <i>Jerusalem</i>	4783	4077	70
56. <i>Adrian</i> built the long wall in <i>Britain</i> — — —	4833	4127	120
57. <i>Constantius</i> defeated the <i>Picts</i> in <i>Britain</i> — — —	5019	4313	306
58. The council of <i>Nice</i> — — —	5038	4332	725
59. The death of <i>Constantine</i> the Great — — —	5050	4344	337
60. The <i>Saxons</i> invited into <i>Britain</i>	5158	4452	445
61. The <i>Arabian Hegira</i> , or flight of <i>Mohammed</i> — — —	5335	4629	662
62. The death of <i>Mohammed</i> —	5343	4637	630
63. The <i>Persian Yesdegird</i> — —	5344	4638	631
64. The art of Printing discovered	6153	5447	1440
65. The Reformation begun by <i>Martin Luther</i> — — —	6230	5524	1517
66. <i>Oliver Cromwell</i> died — —	6371	5665	1658
67. Sir ISAAC NEWTON born at <i>Woolstrop</i> in <i>Lincolnshire</i> , De- cember 25 — — —	6355	5649	1642
— went to <i>Trinity-College</i> in <i>Cam-</i> <i>bridge</i> — — —	6373	5667	1660
— was elected Fellow of that Col- lege — — —	6380	5674	1667
— invented the Fluxions — —	6382	5676	1669
— made Professor of Mathematics, in the room of Dr. <i>Barrow</i> — —	6382	5676	1669
— published his <i>Principia</i> — —	6400	5684	1687
— exerted himself for Religion	6401	5685	1688

	Julian Period.	World's Age.	Since Christ.
Sir ISAAC NEWTON made President of the Royal Society —	6416	5700	1703
— Knighted by Queen ANNE —	6418	5702	1705
— died, March 20 —	6440	5734	1727

In this Table, the years both before and since CHRIST are reckoned exclusive from the year of his birth.

The year of our SAVIOUR'S Crucifixion ascertained; and the darkness at the time of his Crucifixion proved to be supernatural.

Concerning the time of our Saviour's entering upon his public ministry (which may be called the time of his appearance, because, till then, he was not publicly known, so as to be talked of) and also concerning the time of his death, there is a very remarkable prophecy in the IXth chapter of the book of *Daniel*, from the 24th verse to the end; which is in our English Translation as follows:

Ver. 24. *Seventy weeks are determined upon thy people, and upon thy holy city, to finish the transgression, and to make an end of sins, and to make reconciliation for iniquity, and to bring in everlasting righteousness, and to seal up the vision and prophecy, and to anoint the most holy.*

25. *Know therefore and understand, that from the going forth of the command-*

commandment to restore and build Jerusalem, unto the Messiah the prince, shall be seven weeks; and threescore and two weeks the street shall be built again, and the wall, even in troublous times.

26. And after threescore and two weeks shall Messiah be cut off, but not for himself: and the people of the prince that shall come, shall destroy the city and the sanctuary, and the end thereof shall be with a flood, and unto the end of the war desolations are determined.

27. And he shall confirm the covenant with many for one week: and in the midst of the week he shall cause the sacrifices and oblations to cease, and for the overspreading of abomination he shall make it desolate, even until the consummation, and that determined, shall be poured upon the desolate.

In the above translation, one part of the 25th verse is most injudiciously pointed with a semi-colon at *seven weeks*; which ought to run thus, *seven weeks and threescore and two weeks.*

In the 24th verse, what we have rendered

dered *prophecy*, is *prophet* in the original: and in some translations, which I have procured from those who understand the Hebrew very well, instead of *vision and prophecy*, it is rendered *visions and prophets*.

In ver. 27. where we have it *the midst of the week*, all the translations I have procured render it *the half part of the week*; which may be taken either for the first or last half part of it.

In the same verse, where we have it *And he shall confirm the covenant with many for one week*; some translations render it *And in one week a covenant shall be confirmed with many*. Now let the whole be put together agreeable to this translation, without dividing it into different verses (which is only of modern invention) but pointing it here and there for the sake of reading; and it will run thus:

Seventy weeks are determined upon thy people*

* *Seventy sevens*, according to Mr. Purver's translation; which may be reckoned sevens of years as well

people and thy holy city to finish the transgressions and to make an end of sins; and to make reconciliation for iniquity, and to bring in everlasting righteousness, and to seal up the visions and prophets, and to anoint the most holy †. Know therefore, and understand, that from the going forth of the commandment to restore and build Jerusalem, unto the Messiah the prince, shall be seven weeks and threescore and two weeks: the street shall be built again, and the wall even in ‡ troublous times. And after threescore and two weeks shall Messiah be cut off, but not for himself. (And the people of the prince that shall come shall destroy the city and sanctuary, and the end thereof shall be with a flood; and unto the end of the war desolations are determined.) And in one week a covenant shall be confirmed

well as sevens of days: and in the 6th verse of the 4th chapter of *Ezekiel*, we have these remarkable words of God to that Prophet: "I have appointed thee each day for a year."

† Some translate this, *the holy of holies*, and Mr. Purver, *the very holy one*.

‡ By most translators, *in the straitness of times*.

with

with many, and in half part of the week
 HE || *shall abolish the sacrifices and offer-*
ings. And for the overspreading of*
abominations he shall make desolate even
unto consuming; and that which is deter-
mined shall be poured upon the desolate.

'Tis evident, that the first part of this prophecy relates to the coming of Christ; to his being put to death, *not for himself*, but for the sins of mankind, by which great sacrifice he was to put an end to all other sacrifices and offerings; to his introducing the righteousness of ages, and sealing up (or putting an end to) prophecies. And that the latter part mentions the destruction of Jerusalem, in a very striking manner.

In the seventh chapter of *Ezra*, we have an account of a very ample commission (or commandment) which was given by king *Artaxerxes* (who was called *Artaxerxes Longimanus*) to *Ezra*, to go up to Jerusalem, in order to repair that city, and restore the state of the

|| The Messiah.

* *Wing* in the Hebrew.

Jews; and that *Ezra* took his journey on the first day of the first month, viz. the month *Nisan*; which began about the time of the vernal equinox. And on the 14th day of that month (reckoned from the New Moon, at which the month began) the Passover was always kept; for *Josephus** expressly says, “ The passover was kept on the “ 14th day of the month *Nisan*, according to the Moon, when the Sun “ was in Aries.” And the Sun always enters the sign Aries at the time of the vernal equinox.

This commandment was given in the 7th year of *Artaxerxes*’s reign, and *that* year (according to *Ptolemy*’s canon, the rectitude of which was scarce ever called in question) was the 457th year before the year of Christ’s birth: and from the vernal equinox in that year, we are to count the above-mentioned seventy weeks to the death of Christ. For, as the accomplishment of the prophecy must end with the expiation of sin, we cannot suppose these weeks to end at any other time.

* Lib. i. cap. 10.

But,

But, if we count many revolutions of 70 common weeks, from the time of the Jewish passover in the 457th year before the vulgar æra of Christ's birth, we shall find that no Messiah or Saviour did appear on the Earth within that space of time : nor will these reckonings lead us from one Passover to another. And it is certain, from the four Gospels, that Christ was crucified at the time of the Passover ; and St. *John*, chap. xviii. ver. 28. is so particular, as to inform us that our Saviour was crucified on the very day that the Passover was to be eaten by the Jews, who would not defile themselves by mixing with the multitude early in the morning, at the time of his trial.—From these circumstances it is plain that these prophetic weeks mean something very different from the weeks by which we commonly reckon.

In the Old Testament, we read of weeks of years, as well as weeks of days. For, as every seventh day was to be a sabbath for man, on which he was to rest from his labour ; so every seventh

year was to be a sabbath for the land, in which it was to rest from tillage. Let us therefore take these 70 weeks to be weeks of years, making 490 years in all; and the reckoning will lead us from the Passover in the 457th year before the year of Christ's birth to the Passover in the 33d year of our Saviour's age, accounted from the vulgar æra of his birth.

It is expressly foretold in this prophecy, that from the time of the commandment's being given to restore and build Jerusalem, to the Messiah the prince, (or to the time of his appearing in his public character) there should be seven weeks and threescore and two weeks; or 69 weeks in all: the first seven of which, being the straitest or shortest of the times, consisting of 49 years, we may very well allot to the repairing of Jerusalem; after which, there should be threescore and two weeks, or 434 years, to the public appearance of the Messiah: and then there remained only one week, or seven
years,

years, for the public ministry; which, I apprehend, is meant by *confirming the covenant with many*.

But as some of the Translations which I have procured, say, concerning that week, *And in one week a covenant shall be confirmed with many*; and all of them have it, *and in half part of the week* (which might be either the first or last half of it) *HE shall abolish the sacrifices and offerings*; it does not appear that the Messiah is brought in for the whole of the seventieth week, but only for one half of it, *in confirming* (or establishing) *the new and everlasting covenant of the Gospel*; by which, *the righteousness of ages*, mentioned in the first verse of the prophecy, seems to be plainly meant.

And when we consider, that CHRIST's messenger, *John* the Baptist, preached so long before Christ took the public ministry upon himself, as that he acquired great fame in many countries around, which could not be done in a short time, we may believe that the last verse of the prophecy allots the first
half

half of the seventieth week (or three years and an half) to the time of *John's* preaching; at the end of which time he baptized Christ, who was then entering into the thirtieth year of his age (according to St. *Luke*) and then Christ took his public ministry upon himself for the remaining half of the seventieth week; at the end of which he was cut off by the wicked and self-hardened Jews, and so put a *virtual* end to all their sacrifices and offerings; which *finally* ended with the destruction of their city and temple about 37 years after.

So that, in the first place, taking the whole of the prophecy together, as in *ver. 25*, and then dividing it into four different periods or parts as above mentioned; it will very naturally run thus;

From the time of <i>Exra's</i> receiving the commandment to repair <i>Jerusalem</i> , until the expiation of Sin by CHRIST	}	Weeks.	Years.
—		70	or 490
For the time of these repairs	—	7	or 49
From the finishing of these repairs to the coming of CHRIST by his messenger <i>John</i> the Baptist	}	62	or 434
From that time to the end of <i>John's</i> ministry, and the baptism of CHRIST	—	$\frac{1}{2}$	or $3\frac{1}{2}$
From thence to the end of CHRIST's ministry, by his death on the Cross	—	$\frac{1}{2}$	or $3\frac{1}{2}$
In all	—	70	or 490

The beginning of these seventy weeks of years being found to be in the 457th year before Christ's birth, at the time of the Jewish Passover, their ending must have been at the Passover in the 33d year after the year of his birth : and consequently, according to this prophecy, our Saviour was crucified at the end of 490 years after *Exra's* commission.

'Tis plain from all the four Gospels, that the crucifixion was on a Friday ; because it was on the day next before the Jewish sabbath ; and as above mentioned, on the day the Passover was to be eaten (at least) by many of the Jews.

The Jewish year consisted of twelve months, as measured by the Moon, which contains 354 days ; to which they either added 11 days every year, in order to make their years keep pace with the Sun's course of 365 days ; or 30 days in three years. So that, although their months were Lunar, their years were Solar. And they always celebrated the Passover on the fourteenth day of the first Lunar month, reckoning from the

2

first

first time of their seeing the New Moon; which, especially at that time of the year, might be when she was about 24 hours old; and consequently their fourteenth day of the month fell upon the day of Full Moon; and, according to *Josephus*, they always kept the Passover at the time of the Full Moon next after the vernal equinox.

But the Full Moon day on which our Saviour was crucified fell on Friday. And as 12 Lunar months want 11 days of 12 Solar months, the Passover Full Moons (as well as all others) fall 11 days back every year; which being more than a week, by four days, makes it, that, in a few neighbouring years, there cannot be two Passover Full Moons on the same day of the week. And when this anticipation would have made the Passover Full Moon fall before the equinoctial day, they set it a whole month forward, to have it at the first Full Moon after the vernal Equinox; which puts it off the same day of the week again.

The

The dispute among chronologers, about the year of our Saviour's crucifixion, is limited within four or five years at most. And it certainly was in the year in which the Passover Full Moon fell on a Friday.

And I find, by calculation, that the only Passover Full Moon which fell on a Friday, from the 20th year after our Saviour's birth to the 40th, was in the 4746th year of the Julian period; which was the 33d year of his age, reckoning from the beginning of the year next after that of his birth, according to the vulgar *Æra* thereof: and the said Passover Full Moon was on the third day of April.

And thus we have an astronomical demonstration of the truth of this ancient prophecy, seeing that the prophetic year of the Messiah's being cut off was the very same with the astronomical.

Besides, we have the testimony of a heathen author, which agrees with the same year. For *Pblegon* informs us,

C c

that

that in the fourth year of the 202d *Olympiad* (which was the 4746th year of the Julian period, and the 33d year after the year of Christ's birth) there was the greatest eclipse of the Sun that ever was known ; for the darkness lasted three hours in the middle of the day : which could be no other than the darkness on the Crucifixion-day ; as the Sun never was totally hid above four minutes of time, from any part of the Earth, by the interposition of the Moon.

If *Pblegon* had been an astronomer, he would have known that the said darkness could not have been occasioned by any regular eclipse of the Sun ; as the Moon was then in the opposite side of the heavens, on account of her being Full. And as there is no other body than the Moon that ever comes between the Sun and the Earth, it is evident that the darkness at the crucifixion was miraculous, being quite out of the ordinary course of nature.

There

There have been great difficulties about our Saviour's eating the Paschal lamb on the evening of the day before it was eaten by the Jews. But I apprehend this difficulty may be easily removed, when we consider that the Jews began their day in the evening, and ended it in the next following evening. So that, although it was on a different day, according to our way of reckoning, it was still the same day according to theirs. And we do not find that they brought in his eating the lamb on the Thursday evening as any accusation against him : which they would undoubtedly have been glad to do, if they could have made a handle of it for that purpose.

Old Stile.				Hundreds of Years.										
Years less than an Hundred.				0	100	200	300	400	500	600				
				700	800	900	1000	1100	1200	1300				
				1400	1500	1600	1700	1800	1900	2000				
				2100	2200	2300	2400	2500	2600	2700				
				2800	2900	3000	3100	3200	3300	3400				
				3500	3600	3700	3800	3900	4000	4100				
				4200	4300	4400	4500	4600	4700	4800				
				4900	5000	5100	5200	5300	5400	5500				
0	28	56	84	D C	C B	B A	A G	G F	F E	E D				
1	29	57	85	E	D	C	B	A	G	F				
2	30	58	86	F	E	D	C	B	A	G				
3	31	59	87	G	F	E	D	C	B	A				
4	32	60	88	B A	A G	G F	F E	E D	D C	C B				
5	33	61	89	C	B	A	G	F	E	D				
6	34	62	90	D	C	B	A	G	F	E				
7	35	63	91	E	D	C	B	A	G	F				
8	36	64	92	G F	F E	E D	D C	C B	B A	A G				
9	37	65	93	A	G	F	E	D	C	B				
10	38	66	94	B	A	G	F	E	D	C				
11	39	67	95	C	B	A	G	F	E	D				
12	40	68	96	E D	D C	C B	B A	A G	G F	F E				
13	41	69	97	F	E	D	C	B	A	G				
14	42	70	98	G	F	E	D	C	B	A				
15	43	71	99	A	G	F	E	D	C	B				
16	44	72		C B	B A	A G	G F	F E	E D	D C				
17	45	73		D	C	B	A	G	F	E				
18	46	74		E	D	C	B	A	G	F				
19	47	75		F	E	D	C	B	A	G				
20	48	76		A G	G F	F E	E D	D C	C B	B A				
21	49	77		B	A	G	F	E	D	C				
22	50	78		C	B	A	G	F	E	D				
23	51	79		D	C	B	A	G	F	E				
24	52	80		F E	E D	D C	C B	B A	A G	G F				
25	53	81		G	F	E	D	C	B	A				
26	54	82		A	G	F	E	D	C	B				
27	55	83		B	A	G	F	E	D	C				

A Table showing the Dominical Letters for years before the year of Christ's birth.

Old Stile.				Hundreds of Years.							
				0	100	200	300	400	500	600	
Years less than an Hundred.				700	800	900	1000	1100	1200	1300	
				1400	1500	1600	1700	1800	1900	2000	
				2100	2200	2300	2400	2500	2600	2700	
				2800	2900	3000	3100	3200	3300	3400	
				3500	3600	3700	3800	3900	4000	4100	
				4200	4300	4400	4500	4600	4700	4800	
				4900	5000	5100	5200	5300	5400	5500	
0	28	56	84	D C	E D	F E	G F	A G	B A	C B	
1	29	57	85	B	C	D	E	F	G	A	
2	30	58	86	A	B	C	D	E	F	G	
3	31	59	87	G	A	B	C	D	E	F	
4	32	60	88	F E	G F	A G	B A	C B	D C	E D	
5	33	61	89	D	E	F	G	A	B	C	
6	34	62	90	C	D	E	F	G	A	B	
7	35	63	91	B	C	D	E	F	G	A	
8	36	64	92	A G	B A	C B	D C	E D	F E	G F	
9	37	65	93	F	G	A	B	C	D	E	
10	38	66	94	E	F	G	A	B	C	D	
11	39	67	95	D	E	F	G	A	B	C	
12	40	68	96	C B	D C	E D	F E	G F	A G	B A	
13	41	69	97	A	B	C	D	E	F	G	
14	42	70	98	G	A	B	C	D	E	F	
15	43	71	99	F	G	A	B	C	D	E	
16	44	72		E D	F E	G F	A G	B A	C B	D C	
17	45	73		C	D	E	F	G	A	B	
18	46	74		B	C	D	E	F	G	A	
19	47	75		A	B	C	D	E	F	G	
20	48	76		G F	A G	B A	C B	D C	E D	F E	
21	49	77		E	F	G	A	B	C	D	
22	50	78		D	E	F	G	A	B	C	
23	51	79		C	D	E	F	G	A	B	
24	52	80		B A	C B	D C	E D	F E	G F	A G	
25	53	81		G	A	B	C	D	E	F	
26	54	82		F	G	A	B	C	D	E	
27	55	83		E	F	G	A	B	C	D	

A Table showing the Dominical Letters for years after the year of Christ's birth.

*Dominical Letters for the
New Stile.*

1752	B	A	1787	G
1753	G		1788	F E
1754	F		1789	D
1755	E		1790	C
1756	D	C	1791	B
1757	B		1792	A G
1758	A		1793	F
1759	G		1794	E
1760	F	E	1795	D
1761	D		1796	C B
1762	C		1797	A
1763	B		1798	G
1764	A	G	1799	F
1765	F		1800	E
1766	E		1801	D
1767	D		1802	C
1768	C	B	1803	B
1769	A		1804	A G
1770	G		1805	F
1771	F		1806	E
1772	E	D	1807	D
1773	C		1808	C B
1774	B		1809	A
1775	A		1810	G
1776	G	F	1811	F
1777	E		1812	E D
1778	D		1813	C
1779	C		1814	B
1780	B	A	1815	A
1781	G		1816	G F
1782	F		1817	E
1783	E		1818	D
1784	D	C	1819	C
1785	B		1820	B A
1786	A		1821	G

*A Table shewing the Days of the
Months for ever, both in the Old
and New Stile, by the Dominical
Letters.*

Months.	A	B	C	D	E	F	G
Janu.	1	2	3	4	5	6	7
31	8	9	10	11	12	13	14
Octob.	15	16	17	18	19	20	21
31	22	23	24	25	26	27	28
	29	30	31	—	—	—	—
Feb. 28	5	6	7	8	9	10	11
Mar. 31	12	13	14	15	16	17	18
Nov. 30	19	20	21	22	23	24	25
	26	27	28	29	30	31	—
April	2	3	4	5	6	7	8
30	9	10	11	12	13	14	15
July	16	17	18	19	20	21	22
31	23	24	25	26	27	28	29
	30	31	—	—	—	—	—
August	6	7	8	9	10	11	12
31	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31	—	—
Sept.	3	4	5	6	7	8	9
30	10	11	12	13	14	15	16
Dec.	17	18	19	20	21	22	23
31	24	25	26	27	28	29	30
	31	—	—	—	—	—	—
May	7	8	9	10	11	12	13
31	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31	—	—	—
June	4	5	6	7	8	9	10
30	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	—

By the preceding Tables (p. 196, 197, 198) the day of the month answering to any given day of the week, and the day of the week answering to any given day of the month, may be found, in the Old Stile, within the limits of 5500 years before the year of Christ's birth, and 5500 years after it: and, in the New Stile, from *A. D.* 1752, to 1821 inclusive, as follows:

1. For any given year before Christ, look for the complete hundreds of that year (when its number amounts to hundreds) at the head of the Table on page 196, and for the years below or less than an hundred, to make up the number of the given year, at the left hand; and where the columns meet, you have the Dominical letter for the given year. Thus, suppose the Dominical letter was required for the 585th year before the year of Christ 1, which was the 584th before the year of his birth. Under 500 at the head of the Table, and against 84 at the left hand, I find FE, which is the Dominical letter required ;
and

and shews the said year to have been a Leap year ; as every Leap year has two Dominical letters, the first of which serves for January and February, and the last for all the rest of the year. The Dominical letter for any given year after the birth of Christ is found in the same way by the Table in page 197. Thus, suppose it was required for the year 1747 ; I look for 1700 at the head of the Table, and downward thence, in that column against 47 at the left hand, I find D ; which shews that D was the Dominical letter for the year 1747. These two Tables shew the Dominical for the Old Stile : and the Table on page 198 shews it for the New Stile, from *A. D.* 1752 to *A. D.* 1821.

2. Having found the Dominical letter for the given year, look for that letter at the top of the Table shewing the days of the months (page 198) and under the said letter, you have all the days of the months which are Sundays in that year, in the divisions of the months.

Under

Under the next letter toward the right hand, all the days in the column are Mondays; those under the next are Tuesdays; and so on. When you are out at the right hand of the Table, go back to the left, and so reckon on according to the order of the days of the week.

Thus, suppose for the 585th year before Christ, for which the Dominical letter (or letters) was FE; the first serving for January and February, and the last for all the rest of the year; in the Table, pag. 198, I find, under F, the 6th, 13th, 20th and 27th of January; and the 3d, 10th, 17th and 24th of February: and then, under E, I find the 2d, 9th, 16th, 23d and 30th, of March and November; the 5th, 12th, 19th and 26th of October; the 6th, 13th, 20th and 27th of April and July; the 3d, 10th, 17th, 24th and 31st of August; the 7th, 14th, 21st and 28th of September and December; the 4th, 11th, 18th and 25th of May; and the 1st, 8th, 15th, 22d and 29th

D d

of

of June; which being all Sundays in that year, the rest of the days of the months answering to given days of the week, are easily found. For example; if it was required to know on what day of the week the 28th of May fell, in the abovementioned year; I look for the 28th of May in the Table, and I find A stands at the top of the column in which that day is found: and, as the 25th of May fell on Sunday, 'tis plain that the 28th of May must have been on Wednesday.

Again, suppose it was required to find on what day of the week Christmas-day will fall upon in the year 1767, New Stile. The Dominical letter for that year is D. Then, under D in the division for December, in the Table, I find that the 6th, 13th, 20th and 27th are Sundays; and consequently, as the 20th of December falls on Sunday, the 25th (or Christmas-day) must be on Friday. More examples would be superfluous.

How

*How to divide circles and straight lines,
into any given number of equal parts,
whether odd or even.*

When the given number of equal parts, into which a circle, or a straight line is to be divided, are even, and can be divided by 2, 3, 4, &c. the operation is too easy to need any description: but when the given number of parts is odd, as 365, 59, or 31 (which are numbers often wanted) 'tis found difficult to divide them, even by a great many trials with the compasses.

In order to avoid this difficulty I shall shew a method, by which it is as easy to divide either a given circle, or a given straight line, into any odd number of parts, as into any even number; and have all the spaces between the division-lines as equal among themselves as is sufficient for the purpose: provided the operator has a good sector, knows how to open it till the two 60's on the line of chords are as far asunder, when tried

by the compasses, as is equal to the length of the radius, or semidiameter, of the given circle.

There can be no given number of odd divisions or parts, but may have as many subtracted from it as will reduce it to an even number, which may be bisected, trisected, or quartered, &c. And therefore, by finding the length of an arc in the circle that will bear the same proportion to the odd number taken off, as the whole circle bears to the whole given number, this arc may be easily divided into as many equal parts as are contained in the odd number which was subtracted; and then, the remaining number being even, the remaining part of the circle may be easily divided into that number.

All circles contain 360 degrees. Therefore, as the whole number of parts, into which the circle must be divided, is to 360, so is the number of parts subtracted to the number of degrees, and parts of a degree, contained in the arc in which they must be divided.

vided. Thus, suppose it was required to divide a given circle into 365 equal parts: subtract five of these parts, and there will remain 360, which may be first divided into six equal parts, each of these again into six, and each of these last into ten; by which there will be 360 in all. Now say, as 365 parts is to 360 degrees (the whole circle) so is the five parts subtracted to the arc they will fill; which arc, by the calculation, will be found to be 4 degrees, and 93 hundred parts of a degree; which is a little more than 9 tenths.

Therefore, having taken the length of the semidiameter of the given circle by your compasses; open the sector so, as that the two points of the compasses may reach (cross-wise on the sector) from 60 to 60 degrees in the line of the chords; and keeping the sector at that opening, take off 4 degrees and 93 hundredth parts of a degree (as near as you can guess by the eye) cross-wise, from the line of chords, near their beginning at the joint; and set that extent

tent with your compasses upon the periphery of the circle, making marks with the points, and divide the space between the marks into 5 equal parts; and then divide the rest of the circle, first into six equal parts, then each of these again into six, and each of these last into ten; and so you will have the whole circle divided into 365 equal parts, as was required.

Again, suppose a given circle was to be divided into 59 equal parts: subtract 9, and there will remain 50. Then, as 59 parts are to 360 degrees, so are 9 parts to the measure of the arc they will contain; which, by the operation will be found to be 54.91 degrees. Therefore, set off 54.91 (or $54\frac{9}{10}$) degrees upon the circle, and divide that space into 9 equal parts; then divide the rest of the circle, first into 5 equal parts, and then each of these parts into 10; and the whole will be divided into 59 equal parts, as was required. As twice $29\frac{1}{2}$ make 59, this division will do very well for a circle containing the

the $29\frac{1}{2}$ days of the Moon's age from change to change; as the above division of 365 will do for the days in a common year.

Once more; suppose a given circle must be divided into 31 equal parts: subtract 1, and there will remain 30. Then, as 31 parts are to 360 degrees, so is one part to 11.61 degrees (or 11.6) near enough for the purpose. Therefore, set off 11.6 degrees on the circle for the one odd part; and divide the rest of the circle, first into six equal parts, and then each of these parts into five: and the whole circle will be divided into the required number of 31 equal parts. This answers to the divisions of the common month-day circle in clocks.

The method of dividing straight lines of given lengths into any given number of equal parts is the same as above; only, instead of degrees and parts of a degree, we make use of inches and parts of an inch. Thus,

Suppose

Suppose a given straight line was 7 inches and $\frac{3}{10}$ parts of an inch (or 7.3 inches) in length; and it was required to divide that line into 43 equal parts. Subtract 3, and there will remain 40: then, as 43 parts is to 7.3 inches, so is 3 parts to a fourth proportional number, which will be found to be 0.51, or $\frac{51}{100}$ part of an inch. Therefore, from a common scale, where you have an inch diagonally divided into 100 equal parts, take off 51 of these parts in your compasses; and setting one foot in either of the ends of the given line, make a mark with the other foot upon the line; and divide that space into 3 equal parts, and the rest of the line into 40; and you will have the whole line divided into 43 equal parts, as was desired.

By this method, a wheel may be easily divided into any given number of teeth. For, if a slip of paper be laid round the edge of the wheel, so as the ends may just meet; the said slip may be divided into as many equal parts as the wheel must have teeth; and then,

having put a little starch or gum all round the edge of the wheel, put the paper round it again, and divide the wheel by the marks on the paper.

How to find two requisite points in the tube of a Thermometer, and then to divide the scale thereof.

First, put the bulb of the tube into water just freezing, or snow just thawing; and, at that part where the top of the Mercury settles in the tube, make a mark on the tube. Then, if the tube be long enough to contain the rising of the Mercury for boiling water, put the bulb into boiling water; and, at the height to which the boiling raises the Mercury, make a mark on the tube. This done, apply the tube to the intended scale; and against the first mark, place the number 32 on the scale, to denote the freezing point; and against the second mark, place the number 212 on the scale, to denote the boiling point.

E e

point. This done, divide the space between the two marks on the scale into 180 equal parts; and continue these divisions both above the boiling and below the freezing point, for the whole length of the tube upon the scale, and number them accordingly. And then the heats of several bodies may be shewn, by being expressed on the scale, as follows :

Air, in severe cold weather in Britain, from 15 to 25. Air at Midsummer, from 65 to 68. Extreme heat of the Summer-Sun, from 86 to 100. Human heat, about 97. Butter just melting, 95. Alcohol (or pure spirits) boils with 174 or 175, Brandy with 190, water with 212, oil of Turpentine with 550, and Tin melts with 408. Milk freezes about 30, Vinegar about 28, and Blood about 27.

If the tube be not long enough to bear boiling water, first find the freezing point as above directed ; and then put the bulb under your arm-pit, next to your skin, and hold it there as long as
you

you observe the Mercury to rise in the tube, and then make a mark on the tube where the Mercury settles. This done, apply the tube to the intended scale; on which, place 32 at the first mark, for the freezing point, and 97 at the second mark for the degree of human heat. Then divide the space between these two numbers on the scale into 65 equal parts, and continue these divisions above 65 and below 32, as far as the tube goes; and lastly, place the different appellations of heat and cold on the scale as abovementioned.

Rules for finding the Areas or superficial contents of Plane Figures and of solid Bodies.

- I. *The diameter of a circle being given, to find its circumference and its area.*

As 113 is to 355, so is the diameter of the circle, taken in any measure,

E c 2

as

as feet, inches, &c. to its circumference in the like measures.

To find the area, in circular measure, multiply the diameter by itself, and the product will be the area sought.

To find the area in square measure, say, as 1 is to 0.7854 (or rather 0.785399) so is the square of the diameter to the area sought, in such measures as the diameter was taken.

2. *To find the area of the sector of a Circle.*

As 360 degrees is to the degrees in the arc of the sector, so is the area of the whole circle to the area of the sector. Or, multiply the length of the radius of the sector, by the length of the arc of the circle, under which it is contained, and divide the product by 2; the quotient is the area sought.

3. *To find the area of an Ellipsis.*

Multiply the longest diameter by the shortest, and the product by 0.7854;
the

the last product is the area sought, in square measure.

4. *To find the area of a Parabola.*

Multiply the base, or greatest ordinate, by the perpendicular height, and the product by 2 : then divide the last product by 3, and the quotient will be the area required.

5. *To find the area of a Square, or of a Parallelogram.*

Multiply any side of the square by itself, or one of the longest sides of the parallelogram by one of the shortest; and the product will be the area required.

6. *To find the area of a Triangle, the lengths of whose sides are given.*

If the triangle be a right angled one, multiply the base thereof by the perpendicular,

pendicular, and divide the product by 2; the quotient will be the area, in such square measures as the lengths of the base and perpendicular were given.

If the triangle be an oblique one, and have its sides of different lengths, call the longest side the base; and, from the angular point, opposite to the base, draw a right line perpendicular to the base. Then, find the length of the perpendicular, in such measures as the lengths of the sides were given, and multiply the length of the base by the length of the perpendicular, and divide the product by 2: the quotient will be the area of the whole triangle, as was required.

7. To find the area, or superficial content of any Rectilineal Figure.

If the figure be irregular, and consists of many sides and angles, reduce it all into triangles; and then find the content of each triangle by itself, as directed by the foregoing rule. The sum
of

of all these areas or contents, being added together, will be the area or content of the whole figure.

8. *To find the content of a Field that has been surveyed by the chain, and plotted down by a scale of chains and links.*

The common chain for surveying contains 100 links, and the common plotting scale has inches divided by diagonal lines into 100 equal parts; so that an inch answers to a chain in plotting, and the parts to links. 'Tis all the same if half an inch, or a quarter of an inch, be divided into 100 equal parts; for they may be used in plotting: and always the smallest divisions are used when the largest quantities of land are to be plotted down on paper.

The field being plotted down, reduce it all into triangles; and measure the base and perpendicular height of each triangle, by your compasses, on the scale by which the field was plotted, setting down the number of chains and
links

links of each measure as if they were all whole numbers of links. Thus 7 chains, 24 links, are wrote down, 724 links, and 5 chains, 9 links, are wrote down 509 links, and so on. Then multiply the length of the base of each triangle in links, by its perpendicular height in links, and add all the products together into one sum: which done, divide the whole sum by 2, and the quotient will be the area of the field, in square links of the chain.

From the number of square links contained in this area, cut off five figures to the right hand (because 100000 square links make an acre) and what remains to the left hand will be Acres; and those which are cut off will be decimal parts of an Acre.

Multiply these decimals of an Acre by 4, and from the product cut off five figures to the right hand; and what remains to the left will be Roods, as those cut off will be decimals of a Rood.

Lastly, multiply these decimals of a Rood by 40, and cut off five figures of
the

the product to the right hand: and what remains to the left will be Perches, as those cut off will be decimals of a Perch.

And, by this method, the whole content of a field will be had, in acres, roods, and Perches.

9. *The diameter of a Circle being given; to find the side of a square whose area is equal to the area of the Circle: and the reverse.*

As 1 is to 0.8862269, so is the diameter of the circle to the side of the square, whose area is equal to the area of the circle. Or having found the area of the circle by Prob. 1. extract the square root of that area, and it will be the side of the square sought.

To find the diameter of a circle whose area shall be equal to the area of a given square; say, as 1 is to the side of the square, so is 1.128379 to the diameter of the circle required.

F f

10. To

To find the area or superficial contents of a Globe.

Multiply the diameter of the globe by itself, and the product by 3.14159; the last product is the superficial content, in square measure of such parts as the diameter was taken.

The superficies of any globe is equal to four times the area of its greatest circle.

11. *To find the superficial content of a Cylinder.*

As 1 is to the length of the cylinder, so is its circumference to the superficial content of its convexity, to which add the areas, or two flat circular surfaces at each end (found by Prob. 1), and the whole will be the superficial content required.

12. *To find the superficial content of a Cone.*

As 1 is to the oblique (not the perpendicular) height of the cone, so is half

half the circumference of the base to the superficial content of the convexity; to which add the area, or superficial content of the base, and you will have the whole superficial content of the cone, as was required.

13. *To find the superficial content of the Fruustum of a Cone.*

As 1 is to the oblique altitude of the Fruustum, so is half the sum of the circumference, at the top and bottom, to the superficial content of the convex part: to which add the superficial contents of the circular top and base, and you will have the whole superficiality of the Fruustum required.

14. *To find the superficial content of a Prism.*

This may be found in all respects as in the Cylinder, regard being only had to the figure of the base.

F f 2

15. *To*

15. *To find the superficial content of a Pyramid.*

This is done in all respects as the cone, regarding the figure of the base: and consequently the superficial content of the Frustum of a Pyramid may be found in the same manner.

16. *To find the superficial content of any of the five Platonic bodies.*

As 1 is to the side of the given Platonic body,

$$\text{so is } \left\{ \begin{array}{l} 1.73205 \\ 3.46410 \\ 6.00000 \\ 8.66025 \\ 20.64573 \end{array} \right\} \text{To the su-} \quad \left\{ \begin{array}{l} \text{Tetredron.} \\ \text{Octaedron.} \\ \text{Hexaedron.} \\ \text{Icosaedron.} \\ \text{Dodecaedron.} \end{array} \right. \text{perfi-} \quad \left\{ \begin{array}{l} \text{content of} \\ \text{the} \end{array} \right.$$

Or, as 1 is to the square of the side of either of these Platonic bodies, so are the above numbers in this proposition, to the superficial content of the respective Platonic body.

16. *The*

17. *The diameter of a sphere being given, to find the side of any of the Platonic bodies, that may be either inscribed in the sphere, or circumscribed about the sphere, or that is equal to the sphere.*

As 1 is to the number in the following Table, respecting the thing sought, so is the diameter of the given sphere to the side of the Platonic body sought.

The diameter of a sphere being unity, the side of a	That may be inscribed in the sphere, is	That may be circumscribed about the sphere, is	That is equal to the sphere, is
Tetraëdron	0.816497	2.44948	1.64417
Octaëdron	0.707107	1.22474	1.03576
Hexaëdron	0.577350	1.00000	0.88610
Icosaëdron	0.525731	0.66158	0.62153
Dodecaëdron	0.356822	0.44903	0.40883

18. *The side of any of the five Platonic bodies being given, to find the diameter of a sphere that may be inscribed in that body, or circumscribed about it, or that is equal to it.*

As the respective number in the above Table, under the title, *Inscribed*,
Circum-

Circumscribed, Equal, is to 1; so is the side of the given Platonic body, to the diameter of its inscribed, circumscribed, or equal sphere, in solidity.

19. *The side of any of the five Platonic bodies being given, to find the side of either of the Platonic bodies which are equal in solidity to that of the given body.*

As the number under the title *Equal*, against the given Platonic body, is to the number under the same title against the body whose side is sought, so is the side of the given Platonic body to the side of the Platonic body sought.

Rules for finding the solid contents of Bodies.

1. *To find the solid contents of a Sphere or Globe.*

Multiply the diameter of the sphere twice into itself (which is cubing it)
and

and the product by 0.5236; the last product is the solidity required.

2. *To find the solid content of a Spheroid.*

As 14 is to 11, so is the square of the conjugate diameter, multiplied by two thirds of the transverse diameter, to the solid content required.

3. *To find the solid content of a Cube.*

Multiply the side of the cube twice into itself, and the product will be the solid contents thereof.

4. *To find the solid contents of a Parallelopipedon.*

Multiply the length by the breadth, and the product by the depth; the last product will be the solid content required.

5. *To find the solid contents of a Prism.*

Multiply the area of the triangular base by the height of the Prism; and

the product will be the solid contents thereof.

6. *To find the solid contents of a Cone, and also of a Pyramid.*

As 3 is to the area of the base, taken in any measure, so is the perpendicular altitude of the Cone, or of the Pyramid, to its solid contents, in the same measure.

7. *To find the solid contents of the Frustrum of a Cone, in cubic measure.*

Multiply the diameters at top and bottom into one another, and to their product add a third part of the square of their difference: multiply this sum by 0.7854, and the product shall be a mean area; which being multiplied by the perpendicular height, the last product shall be the solid content, in cubic measure, of the whole, in such parts (as inches, feet, &c.) as the diameters and height were taken.

8. *To*

8. *To find the solidity of the Frustrum of a Pyramid.*

This is done in all respects as in the Frustrum of a Cone, only having respect to the figures of its flat base and top, as they may be triangular, square, hexagonal, &c.

9. *To find the solid contents of any of the five Platonic bodies.*

As 1 is to the cube of the side of any of these bodies, so is 0.117851 to the solid contents of the Tetraëdron, 0.417404 to that of the Octaëdron, 1.000000 to that of the Hexaëdron, 2.181695 to that of the Icosaëdron, and 7.663199 to the solid content of the Dodecaëdron.

10. *To find the solid contents of any irregular body, even if it were a Gooseberry bush, provided you have a vessel that will fully hold it.*

Let the vessel be filled quite up to the brim with water, and weighed in a
G g
balance :

balance: then put the irregular body into the vessel, till it be quite covered with the water, and it will cause as much water to run over, as is equal to its whole bulk. This done, take the body out of the water, and then find how much less the vessel weighs, than it did, when full of water, before the body was put into it. Reduce this deficiency of weight into Troy grains, and divide the number of grains by 253.18287 (because the weight of a cubic inch of common water is 253.18287 grains) and the quotient will be the solid contents of the body in cubic inches, which may be reduced to cubic feet by dividing the number of inches by 1728, the number of cubic inches in a cubic foot.

N. B. The outside of the vessel must be wetted when it is full of water, and its weight taken, before the body be put into it; for otherwise, part of the water which the body causes to run over, when it is immersed, will
stick

stick to the outside of the vessel, and thereby give a false conclusion.

II. *To find the solid contents of a Cylinder, in cubic inches.*

As 1 is to 0.7854 (or rather to 0.785399) so is the square of the diameter of the Cylinder, taken in inches, to the number of square inches contained in the area of the base of the Cylinder: which number being multiplied by the height of the Cylinder, taken in inches, gives the solid content thereof in cubic inches.

Now, supposing the Cylinder to be hollow, and these measures to be taken in the inside; we may find how much it will hold, in Ale gallons, Wine gallons, Corn gallons, or Corn bushels, thus:

Divide the content in cubic inches by 282, and the quotient will be the number of Ale gallons; by 231, and the quotient will be the content in Wine gallons; by 268.8, and the quotient

G g 2

will

will be Corn gallons; and by 2150.42, and the quotient will be the content in Corn bushels.

12. *To gauge a common Cask or Barrel.*

Measure the inside diameters of the Cask at the end and middle, and take their difference in inches. Multiply this difference by 0.7, and add the product to the diameter at the end; which will give the mean diameter of the cask, very nearly, as if it were a cylindrical vessel of the same contents with that of the cask: which contents may be found in Ale gallons, Wine gallons, Corn gallons, or Corn bushels, by the foregoing Problem.

13. *To gauge a common Vat.*

Multiply the diameters at the top and bottom (taken in inches) into one another, and to their product add one third part of the square of their difference; then multiply this sum by 0.7854, and

and the product will be a mean area, as if the vessel was cylindrical. Multiply this area by the perpendicular height of the Vat, and the product will give the contents thereof, in the number of cubic inches that it will hold: which number may be reduced into Ale gallons, Wine gallons, Corn gallons, or Corn bushels, as above.

A Table, by which the quantity and weight of water in a cylindrical pipe of any given diameter of bore, and perpendicular height, may be found: and consequently, the power may be known that will be sufficient to raise the water to the top of the pipe, in any pump, or other hydraulic engine.

Feet high.	Diameter of the cylindric bore 1 Inch.		
	Quantity of water in cubic Inches.	Weight of Water in Troy ounces.	In Avoirdupois ounces.
1	9.4247781	4.9712340	5.4541539
2	18.8495562	9.9424680	10.9083078
3	28.2743343	14.9137020	16.3624617
4	37.6991124	19.8849360	21.8166156
5	47.1238905	24.8561700	27.2707695
6	56.5486686	29.8274040	32.7249234
7	65.9734467	34.7986380	38.1790773
8	75.3982248	39.7698720	43.6332312
9	84.8230029	44.7411060	49.0873851

For tens of feet high, remove the decimal points one place forward ; for hundreds of feet, two places ; for thousands, three places ; and so on. Then multiply the sums by the square of the diameter of the given bore, and the products will be the answer.

EXAMPLE.

Qu. *The quantity and weight of water in a cylindrical pipe 85 feet high, and 10 inches diameter. The square of 10 is 100.*

Feet high.	Cubic inches.	Troy ounces.	Avoid. ounces.
80	753.982248	397.698720	436.332312
5	47.123890	24.856170	27.270769
85	801.106138 mult. by 100	422.554890 100	463.603081 100
Anf	80110.613800	42255.489000	46360.308100

Which number (80110.6) of cubic inches being divided by 231, the number of cubic inches in a Wine gallon, gives 342.6 for the number of gallons : and the respective weights (42255.489, and

and 46360.3) being divided, the former by 12, and the latter by 16, give 3521.29 for the number of Troy pounds, and 2897.5 for the number of Avoirdupoise pounds, that the water in the pipe weighs. So much power would be required to balance or support the water in the pipe, and as much more to work the engine as the friction thereof amounts to.

Concerning Pumps.

In all Pumps, the pressure of the column of water, or its weight felt by the working power, when raised to any given height above the surface of the well, is in proportion to the height of the column, considered throughout, as if it were equal in diameter to that part of the bore in which the piston or bucket works.

The advantage or power gained by the handle of the pump is the same as in the common lever; that is, as great as the length from the axis of the handle
to

to its end where the power is applied, exceeds the length of the other part of the handle, from the axis on which it turns, to the pump rod wherein it is fixed, for lifting the piston and water.

In the making of pumps, the diameter of the bore where the bucket works should be proportioned to the height which the pump raises water above the surface of the well, as that a man of ordinary strength might work all pumps equally easy, let their heights be what they will. The annexed Table shews how this may be done, and what quantities of water may be raised in a minute by one man, supposing the handle of the pump to be a lever increasing the power five times.

N. B. In the quarto edition of my book of Lectures, pag. 75, last paragraph, and line 3 of column 1, in page 76, *for* bucket *read* surface of the water in the well.

A Table

Find the given height of the pump, in the first column of the Table; and against it, in the second column, you have the diameter which the bore must be of, in inches and hundredth parts of an inch: and in the third column, you have the quantity of water, in English gallons and pints, that a man of common strength can raise to that height in a minute:

With respect to the power required to work the pump, or the quantity of water discharged thereby, it matters not what the diameter of the bore be in any other part than that wherein the piston or bucket works.

<i>A Table for Pump-makers.</i>		
Height of the pump above the surface of the well.	Diameter of the bore where the Piston works.	Water discharged in a minute in gallons and pints.
Feet.	Inches.	Gal. Pints.
10	6.93	81 6
15	5.65	54 4
20	4.90	40 7
25	4.38	32 6
30	4.00	27 2
35	3.70	23 3
40	3.46	20 3
45	3.27	18 1
50	3.10	16 3
55	2.95	14 7
60	2.84	13 5
65	2.72	12 4
70	2.62	11 5
75	2.53	10 7
80	2.45	10 2
85	2.38	9 5
90	2.31	9 1
95	2.25	8 5
100	2.19	8 1

Troy weight compared with Avoirdupoise weight.

- 175 Troy pounds are equal to 144 Avoirdupoise pounds.
- 175 Troy ounces are equal to 192 Avoirdupoise ounces.
- 1 Troy pound contains 5760 grains; and
- 1 Avoirdupoise pound contains 7000 grains.
- 1 Troy ounce contains 480 grains; and
- 1 Avoirdupoise ounce contains 437.5 grains.
- 1 Avoirdupoise dram contains 27.34375 grains.
- 1 Troy pound = 13 ounces 2.651428576 Avoird. drams; and
- 1 Avoirdupoise pound is equal to 1 pound, 2 ounces, 11 pennyweight, 16 grains Troy.

H h

By

By the following Table, we may find how much of either of these weights is contained in any given number of pounds in the other.

Tr. P.	Avoird. Pounds.	Av. P.	Troy pounds.
1	0.822857142857143	1	1.21527777777778
2	1.645714285714286	2	2.43055555555556
3	2.468571428571429	3	3.64583333333333
4	3.291428571428572	4	4.86111111111111
5	4.114285714285715	5	6.07638888888889
6	4.937142857142857	6	7.29166666666667
7	5.760000000000000	7	8.50694444444444
8	6.582857142857143	8	9.72222222222222
9	7.405714285714286	9	10.93750000000000

For tens of pounds, remove the decimal points one place forward; for hundreds of pounds, two places; for thousand, three places; for tens of thousands, four places; and so on, as in the following Examples.

When any fractions remain in the last sum, reduce them to the known parts of a pound, by the common method of reducing decimals to the known parts of an integer: remembering, that in Troy weight, 12 ounces make a pound, 20 pennyweight make an ounce,

ounce, and 24 grains make a penny-weight : and that, in Avoirdupoise weight, 16 ounces make a pound, and 16 drams make an ounce.

EXAMPLE I.

In 175 Troy pounds, Qu. How many Avoirdupoise pounds ?

Troy.	Avoirdupoise.	
100	82.2857142857143	
70	57.6000000000000	<i>Ans. 144.</i>
5	4.1142857142857	
<hr/>		
175	144.0000000000000	

EXAMPLE II.

In 144 Avoirdupoise pounds, Qu. How many Troy pounds ?

Avo.	Troy.	
100	121.5277777777778	
40	48.6111111111111	<i>Ans. 175,</i>
4	4.8611111111111	
<hr/>		
144	175.0000000000000	

EXAMPLE III.

In 72 Avoirdupoise pounds, Qu. How much Troy weight ?

Avo.	Troy.	
70	85.0694444444444	<i>Answer.</i>
2	2.43055555555556	87.5 pounds,
		viz. 87 pounds
72	87.5000000000000	6 ounces.
P.	P.	

In common practice, 'tis sufficient to take out the decimal parts to five or six figures.

By four weights, viz. 1 pound, 3 pounds, 9 pounds, and 27 Pounds, to weigh 40 pounds; or any number of pounds from 1 to 40.

Pounds.	Scale A.	Scale B.	Pounds.	Scale A.	Sc. B.
1	1	0	21	27,3	9
2	3	1	22	27,3,1	9
3	3	0	23	27	3,1
4	1,3	0	24	27	3
5	9	3,1	25	27,1	3
6	9	3	26	27	1
7	9,1	3	27	27	0
8	9	1	28	27,1	0
9	9	0	29	27,3	1
10	9,1	0	30	27,3	0
11	9,3	1	31	27,3,1	0
12	9,3	0	32	27,9	3,1
13	9,3,1	0	33	27,9	3
14	27	1,3,9	34	27,9,1	3
15	27	3,9	35	27,9	1
16	27,1	3,9	36	27,9	0
17	27	9,1	37	27,9,1	0
18	27	9	38	27,9,3	1
19	27,1	9	39	27,9,3	0
20	27,3	0,1	40	27,9,3,1	0

The

The two columns under *Pounds* express the number of pounds to be weighed: to the right hand of which, the column under *A* shews what weights are to be put into one scale of the balance, and the column under *B* shews what weights are to be put into the other: by which means, the scale *B* will be so much lighter than the scale *A*, as to require a weight to be put into it, equal to the given number of pounds to be weighed, as stated at the left hand, under *Pounds*, against the weights in the scales; and then the balance will be even.

If, to the above four weights, one of 81 pounds be added, you may weigh 121 pounds; or any number from 1 to 121: and if, to these, you add a weight of 243 pounds, you may weigh 364 pounds, or any number from 1 to 364.

A Table

*A Table of the specific gravities of
Bodies.*

A cubic inch of	Troy Weight.			Avoird.		Compa- rative weight.
	oz.	pw.	gr.	oz.	drams.	
Very fine Gold	10	7	4.45	11	5.85	19.639
Standard Gold	9	19	6.06	10	14.88	18.887
Guinea Gold	9	7	17.18	10	4.76	17.793
Quicksilver	7	3	4.39	7	13.16	13.565
Lead	5	19	16.32	6	8.86	11.325
Pure Silver	5	17	0.00	6	6.69	11.090
Standard Silver	5	11	3.25	6	1.55	10.534
Copper	4	14	22.62	5	3.33	9.000
Plate Brass	4	8	2.05	4	13.31	8.344
Cast Brass	4	5	10.76	4	10.08	8.001
Steel	4	2	20.21	4	8.71	7.835
Block Tin	3	17	5.52	4	3.99	7.320
Diamond	1	15	21.07	1	13.35	3.400
Fine Marble	1	8	14.11	1	9.30	2.710
Common Glass	1	7	5.20	1	7.88	2.579
Alabaster	0	19	18.43	1	2.03	1.873
Dry Ivory	0	19	5.83	1	0.89	1.823
Dry Boxwood	0	10	20.77	0	9.54	1.029
Sea Water	0	10	20.51	0	9.51	1.028
Common Water	0	10	13.18	0	9.23	1.000
Red Wine	0	10	11.42	0	9.20	.993
Proof Spirits	0	9	19.73	0	8.62	.931
Pure Spirits	0	9	3.27	0	8.02	.866
Æther	0	7	14.00	0	7.46	.720
Cork	0	2	12.77	0	2.21	.240
Air	0	0	0.28	0	0.009	.001

Take away the decimal points from the numbers in the right-hand column, and reckon them to be whole numbers; and they will shew how many Avoirdupoise ounces are contained in a cubic foot of each of the above bodies in the Table.

A Table

*A Table of the different Velocities
and Forces of the Winds.*

Velocity of the Wind.		Perpen- dicular force on one foot Area, in pounds Avoir- dupoise.	Common appellations of the forces of Winds.
Miles in one Hour.	= Feet in one Second.		
1	2.47	.005	Not perceptible.
2	2.93	.020	} Just perceptible.
3	4.40	.044	
4	5.87	.079	} Gentle pleasant wind.
5	7.33	.112	
10	14.67	.492	} Pleasant brisk Gale.
15	22.00	1.107	
20	29.33	1.968	} Very brisk.
25	36.67	3.075	
30	44.00	4.428	} High Winds.
35	51.33	6.027	
40	58.67	7.872	} Very high.
45	66.00	9.963	
50	73.33	12.300	A storm or tempest.
60	88.00	17.712	A great storm.
80	117.33	31.488	A hurricane.
100	146.70	49.200	A hurricane that tears up trees, and carries buildings, &c. before it.

The force of the Wind is as the square of its velocity.

That the force of the wind is as the square of its velocity, I have often proved by experiments made on my *Whirling Table*.

Directions

Directions for Mill-wrights.

To have a water-mill for grinding corn, in the greatest degree of perfection, the float-boards of the undershot water-wheel, or the buckets of the overshot water-wheel, ought to move with a third part of the velocity where-with the water acts upon them; and when the wheel goes at that rate, the millstone ought to make about 60 revolutions in a minute: for, when it makes but about 45 or 50, it grinds too slow; and when it makes about 70 or 75, it heats the meal too much.

When the wheel turns round with a third part of the velocity of the water, the water has then the greatest power to turn the mill.

On these principles I have calculated the following Table, adapted to a water-wheel 18 feet diameter; which, I think, is a good common size.

In the first column, find the perpendicular height of the fall of water,

in feet ; then, against that height, in the second column, you have the velocity of the water in feet *per* second ; in the third, the velocity of the wheel ; and in the fourth, the number of cogs in the wheel, and staves in the trundle, for causing the millstone to go about 60 times round in a minute.

The Mill-wright's Table.

1	2		3	4		5
Perpen- dicular height of the Fall of Water, in Feet.	Velocity of the Wa- ter <i>per</i> Second.	100 Parts of a Foot. Feet.	Velocity of the Wheel <i>per</i> Second.	100 Parts of a Foot. Feet.	Cogs in the Wheel. Staves in the Trundle.	Number of Turns of the Millstone <i>per</i> Mi- nute, by these Cogs and Staves.
1	8	.02	2	.67	127 6	59.92
2	11	.40	3	.80	105 7	60.00
3	13	.89	4	.63	98 8	60.14
4	16	.04	5	.35	95 9	59.87
5	17	.93	5	.98	85 9	59.84
6	19	.64	6	.55	78 9	60.10
7	21	.21	7	.07	72 9	60.00
8	22	.68	7	.56	67 9	59.67
9	24	.05	8	.02	70 10	59.57
10	25	.35	8	.45	67 10	60.09
11	26	.59	8	.86	64 10	60.16
12	27	.77	9	.26	61 10	59.90
13	28	.91	9	.64	59 10	60.18
14	30	.00	10	.00	50 10	59.36
15	31	.05	10	.35	55 10	60.48
16	32	.07	10	.69	53 10	60.10
17	33	.06	11	.02	51 10	59.67
18	34	.02	11	.34	50 10	60.10
19	34	.95	11	.65	49 10	60.61
20	35	.86	11	.95	47 10	59.50

of

*Of the difference between the apparent
Level and the true.*

When a plumb line hangs freely, it hangs directly toward the center of the earth: and a right line, crossing the direction of the plumb line at right angles, and touching the Earth's surface just below the plummet, is a level at that point of the Earth's surface. But, if this right line be continued from that point, keeping still perpendicular to the plumb line, it will rise above the Earth's surface, because the Earth is of a globular shape: and this rising will be as the square of the distance to which the said right line is produced. That is, however much it rises above the surface at one mile's distance, it will rise four times as much at the distance of two miles; nine times as much at the distance of three miles; sixteen times as much at the distance of four miles; and so on. And therefore, if two levels are taken at two points of the

Earth's surface which are at any considerable distance (as suppose a mile) from each other, the level lines produced will intersect each other at a certain angle : and although either of them, so produced, will appear to be a true level, yet it can be so only at that point of the Earth's surface from which it was produced : not at any other.

The height to which a level line, produced from any given place, rises above any other place, is the height of the apparent level above the true at that other place : the quantity of which height is shewn by the following Table, for all distances within the length of a degree of a great circle upon the Earth's surface. There is a Table of the same sort in Dr. LONG's Astronomy, which differs but two inches from this (which I have computed) in the height of the apparent level above the true, for a whole degree, or 60 geographical miles ; which are longer than 60 English miles by 48840 feet.

By

By the most accurate measures of the length of a degree on the Earth's surface, the whole 360 degrees of the Earth's circumference contain 131630400 feet, or 24930 English miles: which in geographical miles (allowing 60 to a degree of a great circle) make only 21600. So that, a geographical mile contains 6094 feet, which exceeds the length of an English mile by 814.

In the Table, a geographical mile (which I have often thought should be the universal standard length of a mile) is called a *minute*, because it is the 60th part of a degree; and the 60th part of such a mile is called a *second*.

As the surface of water naturally answers to the curvature of the Earth's surface (supposing no hills or eminences thereon) 'tis plain that if a long straight channel was made, so as to have its middle part level at any part of the Earth's surface, and the rest continued out both ways in direction of an apparent level from that place; if water should come in at either end of the

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channel,

channel, it would run to the middle thereof: and, if the channel was all of an equal depth, the water would run over at the middle before the channel could be filled at both its ends. And consequently, if a distant spring appeared by a levelling instrument to be just on a level with the house, the water might be brought in a straight channel from the spring to the house; or in pipes, if there was an intervening valley; because the water will rise in crooked pipes, till its surface at both ends is equally distant from the Earth's center.

A Table

A Table shewing the height of the apparent level above the true, to the 10000th part of an Inch.

Seconds.		Feet.	Inches.		Inches.
1		101	6.8		0.003
2		203	1.6		0.012
3		304	8.4		0.027
4		406	3.2		0.048
5		507	10.0		0.074
6		609	4.8		0.106
7		710	11.6		0.145
8		812	6.4		0.189
9		914	1.2		0.239
10		1015	8.0		0.295
11		1117	2.8		0.357
12		1218	9.6		0.425
13		1320	4.4		0.499
14		1421	11.2		0.579
15		1523	6.0		0.665
16		1625	0.8		0.756
17		1726	7.6		0.854
18		1828	2.4		0.947
19		1929	9.2		1.067
20		2031	4.0		1.182
21		2132	10.8		1.303
22		2234	5.6		1.420
23		2336	0.4		1.563
24		2437	7.2		1.702
25		2539	2.0		1.847
26		2640	8.8		2.001
27		2742	3.6		2.154
28		2843	10.4		2.316
29		2945	5.2		2.485
30		3047	0.0		2.659

If the distance of the object from the place of the spectator be

which measured in a great circle upon the Earth's surface amounts to

the height of the apparent level above the true will be

The Table continued.

Seconds.	Feet.	Inches.	Inches.
31	3148	6.8	2.839
32	3250	1.6	3.026
33	3351	8.4	3.218
34	3453	3.2	3.416
35	3554	10.0	3.619
36	3656	4.8	3.829
37	3757	11.6	4.045
38	3859	6.4	4.267
39	3961	1.2	4.494
40	4062	8.0	4.728
41	4164	2.8	4.967
42	4265	9.6	5.212
43	4367	4.4	5.463
44	4468	11.2	5.720
45	4570	6.0	5.983
46	4672	0.8	6.252
47	4773	7.6	6.527
48	4875	2.4	6.808
49	4976	9.2	7.094
50	5078	4.0	7.387
51	5179	10.8	7.685
52	5281	5.6	7.989
53	5383	0.4	8.300
54	5484	7.2	8.616
55	5586	2.0	8.938
56	5687	8.8	9.266
57	5789	3.6	9.600
58	5890	10.4	9.940
59	5992	5.2	10.285
60	6094	0.0	10.637

If the distance of the object from the place of the spectator be

which measured in a great circle upon the Earth's surface amounts to

the height of the apparent level above the true will be

The

The Table continued.

Minutes.	Feet.	Feet.	Inches.
1	6094	0	10.637
2	12188	3	6.548
3	18282	7	11.732
4	24376	14	2.191
5	30470	22	1.923
6	36564	31	10.929
7	42658	43	5.209
8	48752	56	8.763
9	54846	71	9.591
10	60940	88	7.692
11	67034	107	3.067
12	73128	127	7.716
13	79222	149	10.025
14	85316	173	8.836
15	91410	199	5.307
16	97504	226	11.052
17	103598	256	2.070
18	109692	287	2.362
19	115786	319	11.928
20	121880	354	6.768
21	127974	390	10.882
22	134068	429	0.269
23	140162	468	10.931
24	146256	510	6.866
25	152350	554	0.075
26	158444	599	2.558
27	164538	646	2.315
28	170632	694	11.345
29	176726	745	5.649
30	182820	797	9.928

If the distance of the object from the place of the spectator be

which measured in a great circle upon the Earth's surface amounts to

the height of the apparent level above the true will be

The Table concluded.

Minutes.	Feet.	Feet.	Inches.
31	188914	851	10.080
32	195008	907	8.206
33	210102	965	3.606
34	207196	1024	8.280
35	213290	1085	10.227
36	219384	1148	9.448
37	225478	1213	5.493
38	231572	1279	11.712
39	237666	1348	2.755
40	243760	1418	3.072
41	249854	1490	0.663
42	255948	1563	7.527
43	262042	1638	11.665
44	268136	1716	1.077
45	274230	1794	11.763
46	280324	1875	7.723
47	686418	1958	0.956
48	292512	2042	3.464
49	298606	2128	3.245
50	304700	2216	0.300
51	310794	2305	6.629
52	316888	2396	10.232
53	322982	2489	11.108
54	329076	2584	9.259
55	335170	2681	4.683
56	341264	2779	9.381
57	347358	2879	11.353
58	353452	2981	10.598
59	359546	3085	7.199
60	365640	3191	0.912

If the distance of the object from the place of the spectator be

which measured in a great circle upon the Earth's surface amounts to

the height of the apparent level above the true will be

By

By the preceding Table, the length of an arc, in feet and inches, on the Earth's surface, may be found ; if its measure be known in minutes and seconds of a degree. Thus, suppose the length of the arc be 10 seconds, which is the sixth part of a geographical mile ; its measure is 1015 feet 8 inches : and an arc of one minute of a degree, which is one geographical mile, is found to be 6094 feet.

We may also find how far one can see in a true horizon at sea, when the eye is raised to any given height above the surface of the water. Thus, suppose the eye of an observer on a ship at sea to be 22 feet, 2 inches, above the surface of the sea, he will see to the distance of 30470 feet all around him ; or to the distance of 5 geographical miles : for against 22 feet, 1.923 (which may be esteemed 2) inches, in the right hand column, is 30470 feet, in the middle column ; and 5 minutes or geographical miles in the first.

Again, supposing that a gentleman was to make a canal in a piece of ground which is half a geographical mile long, and appeared to be truly level by the common levelling instrument; this being 30 seconds of a degree, or 3047 feet, in length, the height of the apparent level above the true, at that distance, is 2.659 inches: and so much must the farther end of the ground be sunk, in order to have the water equally near the surface of the ground at both ends of the canal.

Once more; suppose an observer to have his eye close at the surface of the sea, and that he then just sees the top of a mountain in the sea, whose distance he knows to be just 60 geographical miles, or 365640 feet; the perpendicular height of that mountain, above the surface of the sea, is 3191 feet and $\frac{9}{10}$ parts of an inch.

If the distance of the mountain so seen be more than 60 geographical miles, which is without the reach of
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the Table; yet its height may be found by the Table, in the following manner.

Suppose the distance of the mountain to be 90 geographical miles, or a degree and an half, in a great circle upon the Earth: take half the given number of miles or minutes, which is 45; and multiply the height of the apparent level above the true, at that distance, by 4: and the product will give the perpendicular height of the mountain. Thus, against 45 minutes you have the height 1794 feet, 11.763 inches; which multiplied by 4, gives 7917 feet, 11.052 inches, for the height of the mountain above the surface of the sea.

According to the measures in the Table, a degree of a great circle upon the Earth contains $69\frac{1}{4}$ English miles.

Of the Mechanical Powers, and of Friction.

From the most simple machine to the most compound Engine, the power or advantage

advantage gained is always as much as the velocity of the moving power exceeds the velocity of the weight or resistance that is moved; making proper allowance for the friction of the machine or engine. So that, if the working power moves through a space of ten, or an hundred, or a thousand inches, whilst the weight or resistance moves only through the space of one inch; the person who works the machine or engine (supposing it to have no friction) could raise ten, or an hundred, or a thousand times as much weight as he could do by his natural strength without it. But the time that is lost will be always as much as the power that is gained.

The simple machines by which power is gained, are six in number, *viz.* the *Lever*, the *Wheel and Axle*, the *Pullies*, the *Inclined Plane*, the *Wedge*, and the *Screw*. Of these six simple machines, all the most compound engines are made: for we know of no other simple machines by which power can be gained.

I. A lever

1. A lever is a bar laid over any prop that will support the weight upon it. If the prop be under the middle of the lever no advantage is gained by it ; for as fast as a man pushes down one end, the power rises on the other. To gain power by it, the length of the part or arm between the man and the prop must be greater than the length of the part or arm between the prop and the weight. And then, as much power will be gained as the length of the longer arm exceeds the length of the shorter.

2. If an axle turns upon its gudgeons, and is fixt into a wheel ; and if a rope that raises the weight coils round the axle, whilst a man pulls a rope that was put round the wheel ; the power gained will be as much as the diameter of the wheel, added to the diameter of the rope, exceeds the diameter of the axle added to the diameter of the rope that coils round it.

3. In the pullies, the power gained is equal to twice the number of pullies in the lower block, to which the weight

is suspended. So that the power gained is always in proportion to the number of parts of the rope by which the lower block and weight are suspended.

4. In the inclined plane, or half wedge, the power gained is as much as the length of the machine exceeds its thickness at the back, on which the stroke is given by the sledge or mallet that drives the machine.

5. In the wedge, the power gained is as much as the length of both the sides of the wedge, taken together, exceeds the thickness of its back, on which the blow is struck by the hammer or mallet.

6. In the screw, the power gained is as much as the circumference of the circle described by the working power, that turns the screw, exceeds the distance between the threads or spirals of the screw.

In the lever, the friction is nothing. In the wheel and axle it is as small as the diameter of the gudgeons (added to the power required to bend the rope) is less than the diameter of the wheel;
but

but it increases according to the weight with which the axle is charged. The like might be said of the pulleys, if they did not rub against one another, or against the sides of the mortises in the block where they are placed. A new rope of one inch diameter, going over a pulley 3 inches diameter, and pulled with a force equal to 5 pounds, requires a force of 1 pound to bend it; and a rope two inches diameter requires four times as much force. In the inclined plane wedge, and screw, the friction is at least equal to the power, because they will sustain the weight in any position when the power is taken off.

Wood greased or metal oiled, have nearly the same friction; and the smoother they are their friction is the less. Yet metals may be so highly polished, as to have their friction increased by the cohesion of their parts.

Wood slides easier upon the ground in wet weather than in dry; and easier than an equal weight of iron in dry weather: but iron slides easier than

L l

wood

wood in wet weather. Iron or steel running in brass has the least friction of any. Lead makes a great deal of resistance. In wood acting upon wood, grease makes the motion at least twice as easy. Wheel naves greased or tarred go four times as easy as when wet.

Smooth soft wood, moving upon smooth soft wood, has a friction equal to about a third part of the weight. In rough wood, the friction is almost equal to half the weight. In soft wood upon hard, or hard upon soft, the friction is equal to about a fifth part of the weight.

In polished steel moving upon polished steel or pewter, the friction is about a fourth part of the weight: on copper a fifth part, and on brass a sixth part of the weight. Metals of the same sort have more friction than different sorts.

In general, the friction increases in the same proportion with the weight. The friction is also greater with a great-

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er velocity; but not so great, in proportion, as the increase of velocity.

To have the friction of machines as little as possible, they ought to be made of the fewest and simplest parts. The diameters of the wheels and pulleys ought to be large, and the gudgeons of the axles as small as can be consistent with their required strength. The sides of the pulleys ought not to be all over flat, but to have a small rising in the middle, to keep them from rubbing against each other's sides, and against the sides of their mortises, at a distance from their axles. All the cords and ropes ought to be as pliant as possible; and for that end, rubbed with grease. The teeth of the wheels should just fit and fill the openings, so as neither to be squeezed nor shake therein. All the parts which work into or upon one another ought to be smooth, the gudgeons ought just to fill their holes, and the working parts must be greased. The rounds or staves of the trundles may be made to turn about upon iron spindles fixt in the

round end boards, which will take off a great deal of friction.

Let the strength of all the parts be in proportion to the stress they are to bear; so as they may last equally well. He is by no means a perfect mechanic who does not only adjust the strength to the stress, but also contrive all the parts to last so as that one shall not fail before another.

When any motion is to be long continued, contrive the machine so, as that the working power may always move or act one way, if it can be done. For this is better and easier performed than when the motion is interrupted, by the power's being forced to move first one way and then another; because every new change of motion requires a new additional force to effect it, and a body in motion cannot suddenly receive a contrary motion without great violence, and danger of tearing the machine to pieces. But, when the nature of the thing requires that a motion should be suddenly communicated to a body, or
suddenly

suddenly stop; let the force act against some spring, to prevent the machine's being damaged by a sudden jolt.

When a machine is moved by two handles or winches on the ends of an axle; the handles are so placed, as that when one is up the other is down: which is the worst way possible of placing them, save that of their being both up or down together. For, when a man raises a weight by means of turning a winch, he loses half his force when the winch is upward, because he pushes himself as much backward as he pushes the winch forward; and when the handle of the winch is down, directly below the axle, he loses half his force, because the winch pulls him as much toward it as he pulls it toward him: and therefore, the greatest effect of his force on the machine is when he either pulls the winch upward, on the side of the axle next to him, or pushes it downward on the side farthest from him: yet, even in these cases, the pulling force is stronger than the pushing.

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In order to remedy this defect, as much as possible; the handles should be so placed, as to stand at right angles to one another: and then, when there is a man at each handle, the effect of the one man's force will be greatest when the effect of the other man's is least, upon the machine. Whereas, in the common way of placing these handles, when the effect of one man's force is the greatest, the other man's is so too; and when the effect of that man's force is the least, so also is the other's; which is working at the greatest disadvantage possible.

A mechanical way of laying down the Sun's Declination right, against the days of the months, either on a circular or rectilineal Scale.

The Sun's declination is useful on many accounts; and remarkably so in those kinds of Sun-dials which are so constructed, as that they may be set

true, in any place where the Sun shines, without the help of a meridian line ; because, when they are truly level, and have the Sun's declination laid down on their stiles, the dial itself being a circular plate placed on the middle of the stile, and at right angles thereto ; if the dial be turned about till the shadow of the circular plate touches the Sun's declination for the given day, the dial will then be truly placed, and the stile (which will be in the plane of the meridian) will cast a shadow on the true solar time of the day. This is the case in *M. Pardie's* universal dial, which is one of the best I know of ; and which the reader will find particularly described in my *Mechanical Lectures*, sold by *Mr. Cadell*, Bookseller in the Strand, London.

There are Tables near the beginning of this book which shew the quantity of the Sun's declination at the Noon of every day of the second year after Leap year, which is the nearest mean of all the four years. But, as the declination very seldom comes to integral degrees
at

at Noon, it is difficult by these Tables to know at what time of the day the declination will amount to compleat degrees without fractions; and consequently it is difficult to lay down the whole degrees thereof against the proper times of the days of the year, in a scale of months; although every day in the scale should be divided into four equal parts, each whereof contains 6 hours.

To avoid this difficulty, I have calculated the following Table, for shewing the times, to the nearest hour of the day, when the Sun's declination amounts to compleat degrees without fractions. Thus, supposing it was required to find on what days of the year, and at what hours of these days, the Sun's declination was just 9 degrees? Look for 9 in the declination columns, and against it you will find April 12, at 16 hours (reckoned forward from the Noon of the day) August 30th, at 0 hours (or at Noon) October 16th at 3 hours past Noon,

Noon, and Feb. 25th at 1 hour past Noon.

Now, if the 365 days of the year be laid down on a scale, and each day be divided into 4 equal parts thereon, in shorter strokes than those which mark the Noons of the integral days; each subdivision by the shorter strokes will represent 6 hours, and any one may trust to the accuracy of his eye in placing the divisions for the whole degrees of declination at or between these subdivisions of the days, as they are shewn by the Table to be at 0 hours (or Noon) 6 hours, 12, or 18 hours after Noon; or at any time sooner or later.

A Table shewing at what times the Sun's declination is whole degrees, and his Place in the Ecliptic at those times.

Decl. N.	North Declination increases in	Sun's Place in the Ecliptic.	Decl. N.	North Declination decreases in	Sun's Place in the Ecliptic.
Deg.	Mon. D. H.	S. °	Deg.	Mon. D. H.	S. °
0	Mar. 20 5	♈ 0 0	1 1/2	June 21 5	♊ 0 0
1	22 18	2 33	23	July 3 1	11 19
2	25 7	5 2	22	12 2	19 56
3	27 20	7 33	21	18 8	25 56
4	30 10	10 5	20	23 12	♌ 0 52
5	Apr. 2 11	12 39	19	28 2	5 13
6	4 14	15 12	18	Aug. 1 4	9 9
7	7 7	17 49	17	5 0	12 48
8	9 23	20 27	16	8 13	16 14
9	12 16	23 7	15	11 23	19 29
10	15 11	25 50	14	15 5	22 37
11	18 7	28 37	13	18 8	25 38
12	21 6	♉ 1 27	12	21 9	28 33
13	24 7	4 22	11	24 7	♊ 1 23
14	27 8	7 23	10	27 4	4 10
15	30 13	10 31	9	30 0	6 53
16	May 3 23	13 46	8	Sept. 2 8	9 33
17	7 11	17 12	7	4 11	12 11
18	11 7	20 51	6	7 4	14 48
19	15 9	24 47	5	9 18	17 21
20	19 20	29 8	4	12 9	19 55
21	25 1	♊ 4 4	3	15 0	22 27
22	31 7	10 4	2	17 13	24 58
23 1/2	June 9 7	18 41	1	20 3	27 27
1/2	21 5	♋ 0 0	0	22 17	♌ 0 0

The Table concluded.

Decl. S.	South Declination increases in	Sun's Place in the Ecliptic.	Decl. S.	South Declination decreases in	Sun's Place in the Ecliptic.
Deg.	Mon. D. H.	S. °	Deg.	Mon. D. H.	S. °
0	Sept. 22 17	♈ 0 0	$\frac{1}{2}$	Dec 21 9	♏ 0 0
1	25 9	2 33	23	Jan. 1 6	11 19
2	27 20	5 2	22	9 16	19 56
3	30 9	7 33	21	15 14	25 56
4	Oct. 2 23	10 5	20	20 10	♏ 0 52
5	5 13	12 39	19	24 16	5 13
6	8 4	15 12	18	28 13	9 9
7	10 18	17 49	17	Feb. 1 4	12 48
8	13 11	20 27	16	4 13	16 14
9	16 3	23 7	15	7 18	19 29
10	18 21	25 50	14	10 19	22 37
11	21 16	28 37	13	13 20	25 38
12	24 11	♏ 1 27	12	16 17	28 33
13	27 9	4 22	11	19 13	♏ 1 33
14	30 10	7 23	10	22 8	4 10
15	Nov. 2 13	10 31	9	25 1	6 53
16	5 19	13 46	8	27 16	9 33
17	9 5	17 12	7	Mar. 2 7	12 11
18	12 20	20 51	6	4 22	14 48
19	16 17	24 47	5	7 12	17 21
20	21 1	29 8	4	10 2	19 55
21	25 21	♏ 4 4	3	12 14	22 27
22	Dec. 1 19	10 4	2	15 4	24 58
23	10 6	18 41	1	17 15	27 27
$\frac{1}{2}$	21 9	♏ 0 0	0	20 5	♏ 0 0

A Table shewing the Latitudes and Longitudes of a great many remarkable Places; and what the times are at London when it is Noon at those places. N. signifies North Latitude, S. South Latitude; E. East Longitude, W. West Longitude, from the meridian of London: F. Forenoon, and A. Afternoon, at London.

Noon at	Latitude.	Longitude.	Time at London.		
	°	°	H.	M.	
Aberdeen	57 10 N.	1 45 W.	XII	7	A.
Abo	60 30 N.	21 30 E.	X	34	F.
Adrianople	42 00 N.	26 30 E.	X	14	F.
Aleppo	36 30 N.	37 40 E.	IX	29	F.
Algiers	36 40 N.	3 20 E.	XI	47	F.
Amsterdam	52 20 N.	4 30 E.	XI	42	F.
Annapolis Royal	45 00 N.	64 00 W.	IV	16	A.
Archangel	64 34 N.	39 00 E.	IX	24	F.
Astracan	47 00 N.	50 00 E.	VIII	40	F.
Azoph	47 15 N.	44 00 E.	IX	4	F.
Bagdat	33 20 N.	43 00 E.	IX	8	F.
Barcelona	41 20 N.	2 00 E.	XI	52	F.
Basil	47 40 N.	7 40 E.	XI	29	F.
Batavia	6 00 S.	106 00 E.	IV	56	F.
Bencoolen	4 00 S.	101 00 E.	V	16	F.
Berlin	52 33 N.	13 31 E.	XI	6	F.
Bern	57 00 N.	7 20 E.	XI	31	F.
Bologna	44 29 N.	11 26 E.	XI	14	F.
Bombay	18 30 N.	72 00 E.	VII	12	F.
Boston	42 24 N.	71 00 W.	IV	44	A.
Bridge Town	13 00 N.	59 00 W.	III	56	A.
Bristol	51 30 N.	2 40 W.	XII	11	A.
Brussels	51 00 N.	4 6 E.	XI	44	F.
Buda	47 40 N.	19 20 E.	X	43	F.
Buenos Ayres	34 35 S.	58 26 W.	III	54	A.
Cadiz	36 31 N.	5 56 W.	XII	24	A.
Cairo (grand)	30 2 N.	31 31 E.	IX	54	F.

Candy.

The Table continued.

Noon at	Latitude.	Longitude.	Time at London.		
			H.	M.	
Candy, in <i>Ceylon</i>	8 00 N.	79 00 E.	VI	44	F.
Cape good Hope	33 55 S.	18 35 E.	X	46	F.
Canton	23 25 N.	112 30 E.	IV	30	F.
Cape Horn	57 30 S.	80 00 W.	V	20	A.
Carthage	11 00 N.	77 00 W.	V	8	A.
Cayenne	5 00 N.	53 00 W.	III	32	A.
Charles Town	32 30 N.	79 00 W.	V	16	A.
Constantinople	41 00 N.	29 00 E.	X	4	F.
Copenhagen	55 41 N.	12 50 E.	XI	9	F.
Corke	51 40 N.	8 25 W.	XII	34	A.
Cracow	50 10 N.	19 55 E.	X	40	F.
Damascus	33 15 N.	37 20 E.	IX	30	F.
Dantzick	54 21 N.	18 36 E.	X	46	F.
Delly	28 00 N.	78 30 E.	VI	46	F.
Domingo, St.	18 20 N.	70 00 W.	IV	40	A.
Dresden	51 00 N.	13 36 E.	XI	6	F.
Dublin	53 16 N.	6 25 W.	XII	26	A.
Edinburgh	55 58 N.	3 00 W.	XII	12	A.
Erzerum	39 56 N.	48 31 E.	VIII	46	F.
Exeter	50 44 N.	3 40 W.	XII	15	A.
Fez	33 30 N.	6 00 W.	XII	24	A.
Florence	43 46 N.	11 7 E.	XI	16	F.
Geneva	46 12 N.	6 25 E.	XI	34	F.
Genoa	44 30 N.	9 30 E.	XI	22	F.
Glasgow	55 50 N.	4 8 W.	XII	16	A.
Goa	15 31 N.	73 50 E.	VII	5	F.
Hague	52 10 N.	4 00 E.	XI	48	F.
Hamburgh	54 00 N.	9 40 E.	XI	21	F.
Hanover	52 32 N.	9 35 E.	XI	22	F.
Havannah	23 00 N.	84 00 W.	V	36	A.
Helena, St.	16 00 S.	6 00 W.	XII	24	A.
Jago, St. <i>Jamaica</i>	18 20 N.	76 30 W.	V	6	A.
Jago, St. <i>Cuba</i>	20 00 N.	76 31 W.	V	6	A.
James Town	37 30 N.	76 00 W.	V	4	A.
Jerusalem	31 50 N.	35 25 E.	IX	38	F.
Ispahan	32 25 N.	52 55 E.	VIII	28	F.
Kingston, <i>Jamaica</i>	17 30 N.	77 00 W.	V	8	A.
Leghorn	43 30 N.	11 00 E.	XI	16	F.
Leyden	52 12 N.	4 00 E.	XI	44	F.
Lima	12 1 S.	81 34 W.	V	24	A.
Lisbon	38 42 N.	9 25 W.	XII	38	A.

LONDON

The Table continued.

Noon at	Latitude.		Longitude.		Time at London.	
	°	'	°	'	H.	M.
LONDON	51	30	N.	0 00	XII	Noon
Louisbourg	45	54	N.	59 55	III	50 A.
Madrid	40	25	N.	3 50	XII	15 A.
Mahon Port	39	50	N.	4 6	XI	44 F.
Malacca	2	12	N.	102 9	V	12 F.
Malaga	36	40	N.	4 45	XII	19 A.
Mantua	42	20	N.	11 15	XI	16 F.
Mecca	21	20	N.	43 30	IX	6 F.
Mexico, <i>America</i>	20	00	N.	103 35	VI	54 A.
Milan	45	25	N.	9 25	XI	22 F.
Moscow	55	45	N.	37 51	IX	29 F.
Nanking	32	00	N.	118 30	IV	6 F.
Naples	40	51	N.	14 19	XI	3 F.
Norwich	52	40	N.	1 26	XI	54 F.
Nurimburg	49	27	N.	11 9	XI	15 F.
Ormus	27	30	N.	56 00	VIII	16 F.
Osnaburg	52	31	N.	7 40	XI	29 F.
Oxford	51	45	N.	1 15	XI	55 F.
Palermo	38	30	N.	13 00	XI	8 F.
Palmyra	33	00	N.	39 00	IX	24 F.
Panama	9	00	N.	82 00	V	28 A.
Paris	48	50	N.	2 25	XI	50 F.
Pegu	17	30	N.	97 00	V	32 F.
Peking	39	54	N.	116 28	IV	14 F.
Perth	56	25	N.	3 10	XII	13 A.
Petersburg	59	58	N.	30 20	IX	58 F.
Philadelphia	40	50	N.	74 00	IV	56 A.
Plymouth	50	26	N.	4 27	XII	18 A.
Pondicherry	11	56	N.	79 53	VI	40 F.
Porto Bello	9	33	N.	79 50	V	19 A.
Port Royal, <i>Jam.</i>	17	30	N.	77 00	V	8 A.
Portsmouth	50	48	N.	1 6	XII	6 F.
Prague	50	00	N.	14 20	XI	3 F.
Presburg	48	20	N.	17 30	X	50 F.
Quebec	46	55	N.	69 48	IV	39 A.
Rhodes	36	20	N.	28 00	X	8 F.
Rome	41	54	N.	12 30	XI	10 F.
Rotterdam	52	00	N.	4 20	XI	43 F.
Salisbury	51	6	N.	1 15	XII	5 A.
Samarcand	40	00	N.	66 00	VII	36 F.
Scanderoon	36	15	N.	37 00	IX	32 F.

Seville

The Table concluded.

Noon at	Latitude.		Longitude.		Time at London.		
	°		°		H.	M.	
Seville	37	15 N.	6	00 W.	XII	24	A.
Siam	14	18 N.	100	50 E.	V	17	F.
Stockholm	59	20 N.	19	25 E.	X	42	F.
Straßburg	48	35 N.	7	51 E.	XI	29	F.
Surat	21	10 N.	72	25 E.	VII	10	F.
Surinam	6	30 N.	56	00 W.	III	44	A.
Syracuse	37	25 N.	15	5 E.	XI	00	F.
Tobolski	58	12 N.	63	10 E.	VII	27	F.
Toledo	39	50 N.	3	15 W.	XII	13	A.
Tripoli	32	54 N.	13	10 E.	XI	7	F.
Turin	45	5 N.	7	45 E.	XI	29	F.
Valladolid	41	36 N.	4	50 W.	XII	19	A.
Venice	45	25 N.	12	4 E.	XI	12	F.
Vera Cruz, <i>Amer.</i>	18	30 N.	100	00 W.	VI	40	A.
Vienna	48	13 N.	16	27 E.	X	54	F.
Ulm	48	24 N.	10	00 E.	XI	20	F.
Upsal	59	52 N.	17	50 E.	X	49	F.
Uraniburg	55	54 N.	12	51 E.	XI	9	F.
Williamsburg	37	20 N.	76	30 W.	V	6	A.
Worcester	52	15 N.	2	15 E.	XI	51	F.
York, <i>England</i>	54	00 N.	0	50 W.	XII	3	A.
New York, <i>Amer.</i>	41	00 N.	72	30 W.	IV	50	A.
Zell	52	52 N.	10	00 E.	XI	20	F.
Zurick	47	52 N.	8	30 E.	XI	26	F.

Wherever XII is found in this Table, it is to be understood to mean Mid-day, or Noon, at London; and the minutes which follow it are the number of minutes after Mid-day, at London, when it is Noon at the place against which XII is found.

Besides the use of this Table in shewing the Latitudes and Longitudes of a

great number of places, it is useful to those who make Sun-dials : For, if a dial be made for the Meridian of London; and as many places as the artist can find room for, be marked on the dial, against the like hours which front them in the Table; the shadow of the Stile will fall upon these places when it is Noon at them respectively.

And, at any time when the Sun shines on the dial, if the time thereon be counted from the shadow to any given place, it will shew the time then at that place; which time will be before Noon, if the shadow be not come to the place; or Afternoon, if the shadow be past the place; just as much as the interval on the dial is, between the given place and the shadow of the Stile.

If the dial be made for any place whose Longitude is East from the Meridian of London, all the places to be marked upon the dial must be set forwarder than the hours which stand against them in the Table, by four minutes of time for every degree of Longitude

tude that the place has for which the dial is to serve. But if the Longitude of the place for which the dial is made is West from the Meridian of London ; all the places to be marked on the dial must be set backward from the times against which they stand in the Table, by four minutes of time of every degree of the place's Longitude. And then, when the shadow of the Stile falls upon these places, it will be Noon at them ; which will be sooner or later than at the place of the dial, according as the Longitudes of these places are East or West from the place of the dial.

A Table for comparing the English Avoirdupoise pound with the Foreign pound weight.

London Pound	1.0000	Hamburgh	1.0865
Antwerp	1.04	Lisbon	1.135
Amsterdam	1.1111	Leghorn	0.75
Abbeville	1.0989	Norimberg	1.1363
Ancona	0.78	Naples	0.71
Avignon	0.8928	Paris	1.1235
Bordeaux	1.0989	Prague	1.2048
Bologna	0.8	Placentia	0.72
Bruges	1.0204	Rochelle	0.8928
Calabria	0.73	Rome	0.7874
Calais	0.9345	Rouen	1.1089
Dieppe	1.0989	Seville	0.9259
Dantzic	0.862	Tholouse	0.8928
Ferrara	0.75	Turin	0.82
Flanders	0.9433	Venice	1.06
Geneva	1.07	Vienna	1.23
Genoa, gros	0.7		

A Table for comparing the English Foot with Foreign measures, in English Inches.

	Inches.		Inches.
English foot	12.000	English yard	36.000
Amsterdam	11.172	English ell	45.000
Paris	12.788	Scotch ell	45.000
Rheinland	12.362	Paris aune	46.786
Scotch	12.061	Lyons aune	46.570
Dantzic	11.297	Geneva aune	44.760
Swedish	11.692	Amsterdam ell	26.800
Brussels	10.828	Danish ell	24.930
Lyons	13.458	Swedish ell	23.380
Bononian	14.938	Norway ell	24.510
Milan foot	15.631	Seville vara	33.127
Roman palm	8.779	Madrid vara	39.166
Naples palm	10.384	Portugal vara	44.031
		Antwerp	

Antwerp ell	27.170	Portugal cavedo	27.354
Brussels ell	27.260	Old Roman foot	11.632
Bruges ell	27.550	Persian arish	38.364
Bononian brace	25.200	The short pike	} 25.576
Romish brace	30.730	of Constantinople	
Florence brace	22.910	The long pike	27.920

The Weight and Value of Gold and Silver Coins.

A Troy pound of gold is worth 46 pounds 14 shillings and 6 pence : for 44 guineas and an half are coined from each pound at the Mint.

A Troy ounce is worth 3 pounds 17 shillings and 10 pence 2 farthings; and a grain is worth 1 penny $3\frac{7875}{10000}$ farthings, in coinage standard.

A Troy pound of Silver (coinage standard) is worth 3 pounds 2 shillings; an ounce is worth 5 shillings and 2 pence; a pennyweight is worth 3 pence and $\frac{4}{10}$ th parts of a farthing; and a grain is worth about half a farthing.

A Five-moidore piece weighs 1 ounce, 14 pennyweights, 15 grains. A 3 pound 12 sh. piece weighs 18 pennyweight, 12 grains. A Guinea 5 pennyweight, $9\frac{4}{10}$ grains. A Moidore 6

N n 2 penny-

pennyweight, 22 grains; and a Pistole
4 pennyweight, 8 grains.

The proportion of Alloy in coinage.

The standard of sterling silver is 11
ounces 2 pennyweight of pure silver,
and 18 pennyweights of copper.

The standard of sterling gold is 11
Troy ounces of pure gold and 1 ounce
of copper for alloy.

Our gold is of equal fineness with the
Spanish, French, and Flemish; but our
silver coin has less alloy in it than
either French or Dutch.

*Jewish weights reduced to English Troy
weight.*

A Shekel, 9 pennyweight, 2.57
grains. An hundred Shekels, or 3 lb.
9 oz. 10 p. w. 17 grains, make a
Manch: and 50 Manches, or 109 lb.
8 oz. 15 p. w. 10 gr. make a Talent.

Jewish

*Jewish Dry-measure reduced to English
Corn-measure.*

A Cab, $2\frac{5}{6}$ pints. An Omer, $5\frac{5}{10}$ pints. A Seah, 1 peck 1 pint. An Ephah, 3 pecks 3 pints. A Lethech, 16 pecks; and an Homer Choron 32.

*Jewish Liquid-measure reduced to
English.*

A Log, 3 quarters of a pint. A Cab, 3 pints. A Hin, 1 gallon 2 pints. A Seah, 2 gall. 4 pints. A Bath or Ephah, 7 gallons 4 pints. A Coron, or Homer, 75 gallons 5 pints; all in wine measure.

Jewish money reduced to English:

A Gerah, 1.36 d. A Bekah, 1 sh. 1.7 d. A Shekel, 2 sh. 3.37 d. A Mina, 6 l. 16 s. 10.5 d. A Talent of silver, 342 l. 3 s. 9 d. A Shekel of gold, 1 l. 16 s. d. A Talent of gold, 5475 l.

A Table shewing the Interest of any sum of money, from a Million to a Pound, for any number of days, at any rate of Interest.

Sum.	l.	s.	d.	q.	Sum.	l.	s.	d.	q.
1000000	2739	14	6	0.99	1000	2	14	9	2.14
900000	2465	15	0	3.29	900	2	9	3	2.12
800000	2191	15	7	1.59	800	2	3	10	0.11
700000	1917	16	1	3.89	700	1	18	4	1.10
600000	1643	16	8	2.19	600	1	12	10	2.80
500000	1369	17	3	0.49	500	1	7	5	3.70
400000	1095	17	9	2.95	400	1	1	11	0.50
300000	821	18	4	1.09	300	0	16	5	1.40
200000	547	18	10	3.40	200	0	10	11	2.30
100000	273	19	5	1.70	100	0	5	5	3.15
90000	246	11	6	0.32	90	0	4	11	5.71
80000	219	3	6	0.96	80	0	4	4	2.41
70000	191	15	7	1.59	70	0	3	10	0.11
60000	164	7	8	0.22	60	0	3	3	1.81
50000	136	19	8	2.85	50	0	2	8	3.51
40000	109	11	9	1.48	40	0	2	2	1.21
30000	82	3	10	0.11	30	0	1	7	2.90
20000	54	15	10	2.74	20	0	1	1	0.60
10000	27	7	11	1.37	10	0	0	6	2.30
9000	24	13	1	3.23	9	0	0	5	3.67
8000	21	18	4	1.10	8	0	0	5	1.40
7000	19	3	6	2.96	7	0	0	4	2.41
6000	18	8	9	0.82	6	0	0	3	3.76
5000	13	13	11	2.58	5	0	0	3	1.15
4000	10	19	2	0.55	4	0	0	2	2.52
3000	8	4	4	2.41	3	0	0	1	3.80
2000	5	9	7	0.27	2	0	0	1	1.26
1000	2	14	9	2.14	1	0	0	0	2.63

Multiply

Multiply the sum by the number of days, and the product thereof by the rate of Interest *per Cent.* then cut off the two last figures to the right hand, and enter the Table with what remains to the left; against which numbers collected, you have the Interest for the given sum.

EXAMPLE.

Qu. *What is the Interest of 100 l. at 5 per Cent. for 365 days.*

Number of days 365
Multiply by 100 l.

The product is 36500
which multiplied by 5 Rate *per Cent.*

makes 1825 100
Then, in the Table,

		l.	s.	d.	q. parts.
Against {	1000 is	2	14	9	6.14
	800	2	3	10	0.11
	20	0	1	1	0.60
	5	0	0	3	1.15
<hr/>		<hr/>			
1825		Ans ^r .	5	0	0 0.00

Just 5 pounds: and in the same way the interest of any other given number of pounds may be found for any given number of days.

The decimals are 100th parts of a farthing.

A TABLE

A TABLE shewing the *Standard Weight, Value, and comparative View of English Silver Money, from the Time of William the Conqueror, A.D. 1066 to A.D. 1764.*

Dates of the several Mint Indentures.	Standard of the Silver at each Period.		Number of Shillings, &c. in the Pound, or 12 Ounces Troy, of Standard Silver coined at each Period.		Weight of 20 Shillings in reckoning of Standard Silver, at each Period.	Weight of fine Silver contained in 20 Shillings in reckoning, at each Period.						
	Fine Silver	Alloy.	sh.	d.								
A. D.	oz.	pw.	oz.	pw.	sh.	d.	oz.	pw.	gr.	oz.	pw.	gr.
1066	11	2	0	18	21	4	11	5	0	10	8	3
1087	11	2	0	18	20	0	12	0	0	11	2	0
1300	11	2	0	18	20	3	11	17	1	10	19	6
1347	11	2	0	18	22	6	10	13	8	9	17	8
1354	11	2	0	18	25	0	9	12	0	8	17	14
1395												
1402												
1412	11	2	0	18	32	0	7	10	0	6	18	18
1422	11	2	0	18	30	0	8	0	0	7	8	0
1422	11	2	0	18	37	6	6	8	0	5	18	10
1426	11	2	0	18	30	0	8	0	0	7	8	0
1446												
1461												
1464	11	2	0	18	37	6	6	8	0	5	18	10
1482												
1483												
1494	11	2	0	18	40	0	6	0	0	5	11	0
1505												
1509												
1532	11	2	0	18	45	0	5	6	16	4	18	6
1543	10	0	2	0	48	0	5	0	0	4	3	8
1545	6	0	6	0	48	0	5	0	0	2	10	0
1546	4	0	8	0	48	0	5	0	0	1	13	8
1547												
1548												
1549	6	0	6	0	72	0	3	6	16	1	13	8
1551	3	0	9	0	72	0	3	6	16	0	16	16
1553	11	1	0	19	60	0	4	0	0	3	13	16
1560	11	2	0	18	60	0	4	0	0	3	14	0
1583												
1601												
1605	11	2	0	18	62	0	3	17	10	3	11	14
1627												
1661												
1671	11	2	0	18	62	0	3	17	10	3	11	14
1685												
1720												
1764	11	2	0	18	62	0	3	17	10	3	11	14

The Table concluded. — For the Prices of Goods in these different Periods, reckoned according to the then current Silver Money, see the two following Pages.

Dates of the feve ral Mint Indentures.	Value of the same 20 Shillings in reckoning of our pre- sent Money.			Propor- tion of Money at each Pe- riod to that of our pre- sent Mo- ney.	Value of the Ounce of the then standard Silver to that of our present Money.		Value of the Ounce of fine Silver at each Period.		Kings and Queens in these Pe- riods.	
A. D.	l.	s.	d.		s.	d.	s.	d.		
1066	2	18	1 $\frac{1}{2}$	2.9062	5	2	1 11	1 $\frac{1}{8}$	Will. Conq.	
1087	3	2	0	3.1000	5	2	1 9	5 $\frac{3}{8}$	Will. Rufus	
1300	3	1	2 $\frac{3}{4}$	3.0614	5	2	1 9	7 $\frac{3}{8}$	Edward I.	
1347	2	15	1 $\frac{3}{8}$	2.7557	5	2	2 0	3 $\frac{3}{8}$	Edward III.	
1354	}	2	9	7 $\frac{1}{4}$	2.4802	5	2	2 3	Richard II.	
1395										
1402									Henry IV.	
1412	1	18	9	1.9375	5	2	2 10	5 $\frac{3}{8}$	Henry VI.	
1422	2	1	4	2.0666	5	2	2 8	1 $\frac{1}{2}$		
1422	1	13	0 $\frac{3}{4}$	1.6531	5	2	3 4	2 $\frac{1}{2}$		
1426	}	2	1	4	2.0666	5	2	2 8	1 $\frac{1}{2}$	Edward IV.
1446										
1461										
1464	}	1	13	0 $\frac{3}{4}$	1.6531	5	2	3 4	1 $\frac{1}{2}$	Edward V.
1482										
1483										
1494	}	1	11	0	1.5500	5	2	3 7	1 $\frac{1}{4}$	Henry VII.
1505										
1509										
1532	}	1	7	6 $\frac{5}{8}$	1.3776	5	2	4 0	5 $\frac{3}{8}$	Henry VIII.
1543										
1545										
1546	}	0	9	3 $\frac{3}{4}$	0.4656	1	10	3 $\frac{3}{8}$	12 0	Edward VI.
1547										
1548										
1549	0	9	3 $\frac{3}{4}$	0.4656	2	9	1 $\frac{1}{2}$	12 0	Mary I. Elizabeth.	
1551	0	4	7 $\frac{7}{8}$	0.2328	1	4	3 $\frac{3}{4}$	24 0		
1553	1	0	6 $\frac{7}{8}$	1.0286	5	1	3 $\frac{3}{4}$	5 5		
1560	}	1	0	8	1.0333	5	2	5 4		7 $\frac{7}{8}$
1583										
1601										
1605	}	1	0	0	1.0000	5	2	5 7	James I. Charles I. Charles II.	
1627										
1661										
1671										
1685										
1720	}								James II. George I.	
1764										George III.

O o

From

From *A. D.* 1000 to 1066, price of a horse 1*l.* 17*s.* 6*d.*—of an ox 7*s.* 6*d.*—of a cow 6*s.*—a swine 2*s.*—a sheep 1*s.* 3*d.*—wheat, *per* quarter, 1*s.* 6*d.*

From *A. D.* 1066 to 1199, price of a horse 12*s.* 5*d.*—an ox 4*s.* 8 $\frac{1}{4}$ *d.*—a sow 3*s.*—a colt 2*s.* 4 $\frac{1}{2}$ *d.*—a calf 2*s.* 4 $\frac{1}{4}$ *d.*—a sheep 1*s.* 8*d.*—wheat, *per* quarter, 3*s.* 1*d.*

From *A. D.* 1199 to 1307, a horse 1*l.* 11*s.*—an ox 1*l.* 0*s.* 7*d.*—a cow 17*s.* 0 $\frac{1}{2}$ *d.*—a lamb 4*s.*—a heifer 2*s.* 1 $\frac{1}{2}$ *d.*—a sheep 1*s.* 7 $\frac{1}{2}$ *d.*—a goose 1*s.* 0 $\frac{1}{2}$ *d.*—a cock 4 $\frac{1}{2}$ *d.*—a hen 3*d.*—wheat, *per* quarter, 1*l.* 3*s.* 2 $\frac{1}{2}$ *d.*

From *A. D.* 1307 to 1418, an ox 2*l.* 6*s.* 1*d.*—a horse 18*s.* 4*d.*—a cow 7*s.* 2*d.*—a calf 4*s.* 2*d.*—a sheep 2*s.* 7*d.*—a goose 9*d.*—a cock 3 $\frac{3}{4}$ *d.*—a hen 2 $\frac{3}{4}$ *d.*—ale, *per* gallon, 7 $\frac{3}{4}$ *d.*—wages of a common day-labourer 4 $\frac{1}{4}$ *d.*—wheat, *per* quarter, 15*s.*

From *A. D.* 1418 to 1524, a horse 2*l.* 4*s.*—an ox 1*l.* 15*s.* 8 $\frac{1}{2}$ *d.*—a cow 15*s.* 6*d.*—a colt 7*s.* 8*d.*—a sheep 5*s.*

a hog 5 s.—a calf 4 s. 1 d.—a cock 3 d.
 a hen 2 d.—day-labourer's wages $3\frac{3}{4}$ d.
 —ale, *per* gallon, $2\frac{3}{4}$ d.—wheat, *per*
 quarter, 11 s. 3 d.

From *A. D.* 1524 to 1604, price of
 a horse uncertain—of an ox 1 l. 16 s.
 7 d.—a sheep 4 s. $3\frac{3}{4}$ d.—a lamb 1 s.
 $4\frac{1}{4}$ d.—a goose 1 s.—beef, *per* stone,
 11 d.—wheat, *per* quarter, 15 s.

From *A. D.* 1624 to 1646, a goose,
 2 s.—a pullet 1 s. 6 d.—a turkey 3 s.
 9 d.—a pheasant 5 s. 6 d.—a partridge
 1 s.—a pigeon 6 d.—wheat, *per* quarter,
 1 l. 19 s. $3\frac{1}{4}$ d.

From *A. D.* 1730 to 1760, price
 of a horse 10 l.—an ox 8 l.—a cow 7 l.
 7 s.—a hog 1 l. 15 s.—a sheep 1 l. 6 s.—
 a turkey 4 s.—a cock 1 s. 3 d.



T A B L E S

S H E W I N G

The present Value of ANNUITIES,

A T

The most common Rates of INTEREST.

By MR. DE MOIVRE.

T A B L E

TABLE I.

The present Value of an Annuity of one Pound, for any number of years not exceeding 100, Interest at 3 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9709	26	17.8768	51	25.9512	76	29.8076
2	1.9135	27	18.3270	52	26.1662	77	29.9103
3	2.8286	28	18.7641	53	26.3750	78	30.0100
4	3.7170	29	19.1884	54	26.5777	79	30.1068
5	4.5797	30	19.6004	55	26.7744	80	30.2008
6	5.4172	31	20.0004	56	26.9655	81	30.2920
7	6.2303	32	20.3887	57	27.1509	82	30.3806
8	7.0197	33	20.7658	58	27.3310	83	30.4666
9	7.7861	34	21.1318	59	27.5015	84	30.5501
10	8.5302	35	21.4872	60	27.6756	85	30.6311
11	9.2526	36	21.8323	61	27.8404	86	30.7099
12	9.9540	37	22.1672	62	28.0003	87	30.7863
13	10.6350	38	22.4925	63	28.1557	88	30.8605
14	11.2961	39	22.8082	64	28.3065	89	30.9325
15	11.9379	40	23.1148	65	28.4529	90	31.0024
16	12.5611	41	23.4124	66	28.7950	91	31.0703
17	13.1611	42	23.7014	67	28.7330	92	31.1362
18	13.7535	43	23.9819	68	28.8670	93	31.2001
19	14.3238	44	24.2543	69	28.9971	94	31.2622
20	14.8775	45	24.5187	70	29.1234	95	31.3224
21	15.4150	46	24.7754	71	29.2460	96	31.3809
22	15.9369	47	25.0247	72	29.3651	97	31.4377
23	16.4436	48	25.2667	73	29.4807	98	31.4928
24	16.9355	49	25.5017	74	29.5929	99	31.5463
25	17.4131	50	25.7298	75	29.7018	100	31.5984

The Value of the Perpetuity is $33\frac{1}{3}$ Years Purchase.

TABLE II.

The present Value of an Annuity of one Pound, for any number of years not exceeding 100, Interest at $3\frac{1}{2}$ per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9662	26	16.8904	51	23.6286	76	26.4709
2	1.8997	27	17.2854	52	23.7958	77	26.5506
3	2.8016	28	17.6670	53	23.9573	78	26.6190
4	3.6731	29	18.0358	54	24.1133	79	26.6850
5	4.5151	30	18.3920	55	24.2641	80	26.7488
6	5.3286	31	18.7363	56	24.4097	81	26.8104
7	6.1145	32	19.0689	57	24.5504	82	26.8700
8	6.8740	33	19.3902	58	24.6864	83	26.9275
9	7.6077	34	19.7007	59	24.8178	84	27.9831
10	8.3166	35	20.0007	60	24.9447	85	27.0368
11	9.0015	36	20.2905	61	25.0674	86	27.0887
12	9.6633	37	20.5705	62	25.1859	87	27.1388
13	10.3027	38	20.8411	63	25.3004	88	27.1873
14	10.9205	39	21.1025	64	25.4110	89	27.2341
15	11.5174	40	21.3551	65	25.5178	90	27.2793
16	12.0941	41	21.5991	66	25.6211	91	27.3230
17	12.6513	42	21.8349	67	25.7209	92	27.3652
18	13.1897	43	22.0627	68	25.8173	93	27.4060
19	13.7098	44	22.2828	69	25.9104	94	27.4454
20	14.2124	45	22.4955	70	26.0004	95	27.4835
21	14.6980	46	22.7009	71	26.0873	96	27.5203
22	15.1671	47	22.8994	72	26.1713	97	27.5558
23	15.6204	48	23.0912	73	26.2525	98	27.5902
24	16.0584	49	23.2766	74	26.3309	99	27.6234
25	16.4815	50	23.4556	75	26.4067	100	27.6554

The Value of the Perpetuity is $28\frac{1}{7}$ Years Purchase.

TABLE

TABLE III.

The present Value of an Annuity of one Pound, for any number of years under 100, Interest at 4 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9515	26	15.9827	51	21.6714	76	23.7311
2	1.8860	27	15.3295	52	21.7475	77	23.7799
3	2.7750	28	16.6630	53	21.8726	78	23.8268
4	3.6298	29	16.9837	54	21.9929	79	23.8720
5	4.4518	30	17.2920	55	22.1086	80	23.9153
6	5.2421	31	17.5884	56	22.2198	81	23.9571
7	6.0020	32	17.8775	57	22.3267	82	23.9972
8	6.7327	33	18.1476	58	22.4295	83	24.0357
9	7.4353	34	18.4111	59	22.5284	84	24.0728
10	8.1108	35	18.6646	60	22.6234	85	24.1085
11	8.7604	36	18.9082	61	22.7148	86	24.1428
12	9.2850	37	19.1425	62	22.8027	87	24.1757
13	9.9856	38	19.3678	63	22.8872	88	24.2074
14	10.5631	39	19.5844	64	22.9685	89	24.2379
15	11.1183	40	19.7927	65	23.0466	90	24.2672
16	11.6522	41	19.9930	66	23.1218	91	24.2954
17	12.1656	42	20.1856	67	23.1940	92	24.3225
18	12.6592	43	20.3707	68	23.2635	93	24.3486
19	13.1339	44	20.5488	69	23.3302	94	24.3736
20	13.5903	45	20.7200	70	23.3945	95	24.3977
21	14.0291	46	20.8846	71	23.4562	96	24.4209
22	14.4511	47	21.0429	72	23.5156	97	24.4431
23	14.8568	48	21.1951	73	23.5727	98	24.4646
24	15.2469	49	21.3414	74	23.6276	99	24.4851
25	15.6220	50	21.4821	75	23.6804	100	24.5049

TABLE I.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at 3 per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	15.05	26	17.50	51	12.26	76	4.05
2	16.62	27	17.33	52	12.00	77	3.63
3	17.83	28	17.16	53	11.73	78	3.21
4	18.46	29	16.98	54	11.46	79	2.78
5	18.90	30	16.80	55	11.18	80	2.34
6	19.33	31	16.62	56	10.90	81	1.89
7	19.60	32	16.44	57	10.61	82	1.43
8	19.74	33	16.25	58	10.32	83	0.96
9	19.87	34	16.06	59	10.03	84	0.49
10	19.87	35	15.86	60	9.73	85	0.00
11	19.74	36	15.67	61	9.42	86	0.00
12	19.60	37	15.46	62	9.11		
13	19.47	38	15.26	63	8.79		
14	19.33	39	15.05	64	8.46		
15	19.19	40	14.84	65	8.13		
16	19.05	41	14.63	66	7.79		
17	18.90	42	14.41	67	7.45		
18	18.76	43	14.19	68	7.10		
19	18.61	44	13.96	69	6.75		
20	18.46	45	13.73	70	6.38		
21	18.30	46	13.49	71	6.01		
22	18.15	47	13.25	72	5.63		
23	17.99	48	13.01	73	5.25		
24	17.83	49	12.76	74	4.85		
25	17.66	50	12.51	75	4.45		

TABLE II.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at $3\frac{1}{2}$ per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	14.16	26	16.28	51	11.69	76	3.98
2	15.53	27	16.13	52	11.45	77	3.57
3	16.56	28	15.98	53	11.20	78	3.16
4	17.09	29	15.83	54	10.95	79	2.74
5	17.46	30	15.68	55	10.69	80	2.31
6	17.82	31	15.53	56	10.44	81	1.87
7	18.05	32	15.37	57	10.18	82	1.42
8	18.16	33	15.21	58	9.91	83	0.95
9	18.27	34	15.05	59	9.64	84	0.48
10	18.27	35	14.89	60	9.36	85	0.00
11	18.16	36	14.71	61	9.08	86	0.00
12	18.05	37	14.52	62	8.79		
13	17.94	38	14.34	63	8.49		
14	17.82	39	14.16	64	8.19		
15	17.71	40	13.98	65	7.88		
16	17.59	41	13.79	66	7.56		
17	17.46	42	13.59	67	7.24		
18	17.33	43	13.40	68	6.91		
19	17.21	44	13.20	69	6.57		
20	17.09	45	12.99	70	6.22		
21	16.90	46	12.78	71	5.87		
22	16.83	47	12.7	72	5.51		
23	16.69	48	12.36	73	5.14		
24	16.56	49	12.14	74	4.77		
25	16.42	50	11.92	75	4.38		

TABLE III.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at 4 per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	13.36	26	15.19	51	11.13	76	3.91
2	14.54	27	15.06	52	10.92	77	3.52
3	15.43	28	14.94	53	10.70	78	3.11
4	15.89	29	14.81	54	10.47	79	2.70
5	16.21	30	14.68	55	10.24	80	2.28
6	16.50	31	14.54	56	10.71	81	1.85
7	16.64	32	14.41	57	9.57	82	1.40
8	16.79	33	14.27	58	9.22	83	0.95
9	16.88	34	14.12	59	9.07	84	0.48
10	16.88	35	13.98	60	9.71	85	0.00
11	16.79	36	13.82	61	8.45	86	0.00
12	16.64	37	13.07	62	8.28		
13	16.60	38	13.52	63	8.90		
14	16.50	39	13.36	64	7.92		
15	16.41	40	13.20	65	7.63		
16	16.31	41	13.02	66	7.33		
17	16.21	42	12.85	67	7.02		
18	16.10	43	12.68	68	6.71		
19	15.99	44	12.50	69	6.39		
20	15.89	45	12.32	70	6.06		
21	15.78	46	12.13	71	5.72		
22	15.67	47	11.94	72	5.38		
23	15.55	48	11.74	73	5.02		
24	15.43	49	11.54	74	4.66		
25	15.31	50	11.34	75	4.29		

Besides the use of the first three of these Tables, as expressed by their titles, they serve likewise to resolve the questions concerning *compound Interest*: as

1. *To find the present Value of 1000 l. payable 7 years hence, at $3\frac{1}{2}$ per Cent.* From the present value of an Annuity of 1 l. certain for 7 years, which, in Table II. is 6.1145, I subtract the like value for 6 years, which is 5.3286; and the remainder .7859 is the value of the 7th year's rent, or of 1 l. payable after 7 years; which multiplied by 1000 gives the answer 785 l. 18 s.

2. If it be asked, *what will be the Amount of the sum S in 7 years at $3\frac{1}{2}$ per Cent.?* Having found .7859 as above, 'tis plain the amount will be $\frac{S}{.7859}$.

3. If the question is, *In what time a sum S will be doubled, tripled, or increased, in any given Ratio at 3, $3\frac{1}{2}$, &c. per Cent.* I take in the proper Table two contiguous numbers, whose difference is nearest

nearest the reciprocal of the *Ratio* given, as $\frac{1}{2}$, $\frac{1}{3}$, &c. And the year against the higher number is the Answer.

Thus, in *Table I.* against the years 22, 23, stand the numbers 15.9639 and 16.4436; whose difference .5067 being a little more than .5, or $\frac{1}{2}$, shews that in 23 Years, a sum *S* will be a little less than doubled, at 3 *per Cent.* compound Interest. And against the years 36 and 37 are 21.8323, and 22.1672; the difference whereof being .3349, nearly $\frac{1}{3}$, shews that in 14 years more it will be almost tripled.

If more exactness is required, take the adjoining difference, whose error is contrary to that of the difference found; and thence compute the proportional part to be added or subtracted. Thus in the last of these Examples, the difference between the years 37 and 38 is .3252, which wants .0081 of .3333 ($= \frac{1}{3}$), as the other difference .3349 exceeded it by .0016. The 38th year is therefore to be divided in the Ratio of 16 to 81; that is $\frac{16}{97}$ of a
 9 year,

year, or about 2 months to be added to the 37 years.

I. To find at what Rate of Interest I ought to lay out a sum S , so as it may increase $\frac{1}{3}$ for Instance, or become $\frac{4}{3} S$ in 7 years. Here the fraction I am to look for among the differences is $\frac{3}{4}$, or the decimal .75, which is not to be found in *Tab. I.* or *II.* till after the limited time of 7 years. But, in *Tab. III.* the numbers against 6 and 7 years give the difference .7599; and the Rate is 4 per Cent. nearly.

So far Mr. DE MOIVRE on this Subject.

In questions concerning the Values of Lives any how combined, recourse must be had to Mr. *De Moivre's* last Edition of his Treatise on Annuities.

The four following Tables, and the Remarks on them, are also copied from Mr. DE MOIVRE's Book on the Doctrine of Chances.

The

The Probabilities of Human Life, according to different Authors.

TABLE I.

By Dr. Halley.

Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.
1	1000	23	580	45	397	67	172
2	855	24	574	46	387	68	162
3	798	25	567*	47	377	69	152
4	760	26	560	48	367	70	142
5	732	27	553	49	357	71	131*
6	710	28	546	50	346*	72	120
7	692	29	539	51	335	73	109
8	680	30	531*	52	324	74	98
9	670	31	523	53	313	75	88*
10	661	32	515	54	302	76	78
11	653	33	507	55	292*	77	68
12	646	34	499	56	282	78	58
13	640*	35	490*	57	272	79	49*
14	634	36	481	58	262	80	41
15	628	37	472	59	252	81	34
16	622	38	463	60	242	82	28
17	616	39	454	61	232	83	23
18	610	40	445	62	222	84	19
19	604	41	436	63	212	*	*
20	598	42	427	64	202		
21	592	43	417*	65	192		
22	586	44	407	66	182		

TABLE

TABLE II.

By Mr. Kerffseboom.

Age. o	Living. 1400	Age.	Living.	Age.	Living.	Age.	Living.
1	1125	26	760	51	495	76	160
2	1075	27	747	52	482	77	145
3	1030	28	735	53	470	78	130
4	993	29	723	54	458	79	115
5	964	30	711	55	446	80	100
6	947	31	699	56	434	81	87
7	930	32	687	57	421	82	75
8	913	33	675	58	408	83	64
9	904	34	665	59	395	84	55
10	895	35	655	60	382	85	45
11	886	36	645	61	369	86	36
12	878	37	635	62	356	87	28
13	870	38	625	63	343	88	21
14	863	39	615	64	329	89	15
15	856	40	605	65	315	90	10
16	849	41	596	66	301	91	7
17	842	42	587	67	287	92	5
18	835	43	578	68	273	93	3
19	826	44	569	69	259	94	2
20	817	45	560	70	245	95	1
21	808	46	550	71	231	96	0.6
22	800	47	540	72	217	97	0.5
23	792	48	530	73	203	98	0.4
24	783	49	518	74	189	99	0.2
25	722	50	507	75	175	100	0.0

TABLE

TABLE III.

By M. de Parcieux.

Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.
1	****	26	766	51	571	76	192
2	****	27	758	52	560	77	173
3	1000	28	750	53	549	78	154
4	970	29	742	54	538	79	136
5	948	30	734	55	526	80	118
6	930	31	726	56	514	81	101
7	915	32	718	57	502	82	85
8	902	33	710	58	489	83	71
9	890	34	702	59	476	84	59
10	880	35	694	60	463	85	48
11	872	36	686	61	450	86	38
12	866	37	678	62	437	87	29
13	860	38	671	63	423	88	22
14	854	39	664	64	409	89	16
15	848	40	657	65	395	90	11
16	842	41	650	66	380	91	7
17	835	42	643	67	364	92	4
18	828	43	636	68	347	93	2
19	821	44	629	69	329	94	1
20	814	45	622	70	310	95	0
21	806	46	615	71	291	96	*
22	798	47	607	72	271	97	*
23	790	48	599	73	251	98	
24	782	49	590	74	231	99	
25	774	50	581	75	211	100	

TABLE IV.

By Messrs. *Smart* and *Simpson*.

Age.	Liv- ing.	Age.	Liv- ing.	Age.	Liv- ing.	Age.	Liv- ing.	Age.	Liv- ing.
1	1280 }	17	480	33	358	49	212	65	99
2	870 }	18	474	34	349	50	204	66	93
3	700	19	468	35	340	51	196	67	87
4	635	20	462	36	331	52	188	68	81
5	600	21	455	37	322	53	180	69	75
6	580	22	448	38	313	54	172	70	69
7	564	23	441	39	304	55	165	71	64
8	551	24	434	40	294	56	158	72	59
9	541	25	426	41	284	57	151	73	54
10	532	26	418	42	274	58	144	74	49
11	524	27	410	43	264	59	137	75	45
12	517	28	402	44	255	60	130	76	41
13	510	29	394	45	246	61	123	77	38
14	504	30	385	46	237	62	117	78	35
15	498	31	376	47	228	63	111	79	32
16	492	32	367	48	220	64	105	80	29
	486								

Remarks on these four Tables of the Probabilities of Human Life.

The first Table is that of Dr. *Halley*, composed from the Bills of Mortality of the city of *Breslaw*; the best, perhaps, as well as the first of its kind: and which will always do honour to the judgment

judgment and sagacity of its excellent author.

The next is a Table of the ingenious Mr. *Kerſſeboom*, founded chiefly upon Registers of the *Dutch* Annuitants, carefully examined and compared for more than a century backward. And *Monſieur de Parcieux*, by a like uſe of the liſts of the *French Tontines*, or *long Annuities*, has furniſhed us Table III; whoſe numbers were likewiſe verified upon the *Necrologies* or mortuary Registers of ſeveral religious houſes of both ſexes.

To theſe is added a Table of Meſſieurs *Smart* and *Simpſon* adapted particularly to the city of *London*; whoſe inhabitants, for reaſons too well known, are ſhorter lived than the reſt of mankind.

Each of theſe Tables may have its particular uſe: the ſecond or third in valuing the *better* ſort of lives, upon which one would chooſe to hold an Annuity; the *French* may ſerve for *London*, or for lives ſuch as thoſe of its inhabitants may be ſuppoſed to be:

Qq 2

while

while Dr. *Halley's* numbers, falling between the two extremes, seem to approach nearer to the general course of nature. And in cases of combined lives, two or more of the Tables may perhaps be usefully employed.

Besides these, the celebrated Monsieur de *Buffon* * has lately given us a new Table, from the actual observations of Monsieur du *Pré de St Maur* of the *French Academy*. This Gentleman, in order to strike a just *mean*, takes three populous parishes in the city of *Paris*, and so many country villages as furnish him nearly an equal number of lives: and his care and accuracy in that performance has been such as to merit the high approbation of the learned editor. It was therefore proposed to add this Table to the rest; after having cleared its numbers of the inequalities that necessarily happen in fortuitous things, as well as those arising from the careless manner in which *Ages* are given to the Parish Clerks; by which the years that

* *Histoire Naturelle*, Tom. II.

are multiples of 10 are generally overloaded.

But this having been done with all due care, and the whole reduced to Dr. *Halley's* denomination of 1000 infants of a year old, there resulted only a mutual confirmation of the two Tables; Mr. *du Pré's* Table making the lives somewhat better as far as 39 years, and thence a small matter worse than they are by Dr. *Halley's*.

We may therefore retain this last as no bad standard for mankind in *general*; till a better police, in this and other nations, shall furnish the proper *Data* for correcting it, and for expressing the Decrements of life more accurately, and in larger numbers.

A Table;

A Table; the first part whereof shews the height to which a Barometer must be raised above the plane surface of the Earth, in order that the Mercury may stand at any given height in the Tube; and the second part shews at what height the Mercury will stand in the Tube, when the Barometer is raised to any given height above the Earth's plane surface.

Part I.			Part II.	
Height of the Mercury in inches	Height of the Barometer in feet above the Earth's plane surface.		Height of the Barometer above the Earth.	Height of the Mercury in inches.
30.000	Feet 0	0	Feet 0	30.00
29.000	915		1000	28.91
28.000	1862		2000	27.86
27.000	2844		3000	26.85
26.000	3863		4000	25.87
25.000	4922		5000	24.93
20.000	10947		Miles 1	24.67
15.000	18715		2	20.29
10.000	29662		3	16.68
5.000	48378		4	13.72
1.000	91831		5	11.28
0.5	110547		10	4.24
0.25	129262		20	1.60
0.1	29 miles, or 153120		25	0.95
0.001	41 miles, or 216480		30	0.23
0.000	53 miles, or 279840		40	0.08

By

By the first part of this Table, and a common Barometer or Weather-glass, the perpendicular height of a hill above the plane surface of the Earth, may be nearly found. Thus, suppose the Mercury was observed to stand at 30 inches in the tube when at the foot of the hill, and at 27 inches when carried up to the top: against this sinking of three inches, you have 2844 feet (or 948 yards) for the perpendicular height of the hill. The second part is too plain to need any description or example.

An account of M. Villette's concave burning Mirror.

This Mirror is 3 feet 11 inches in diameter, and its focal distance is 3 feet 2 inches. It is made of copper, tin, and bismuth.

The effect of the Sun-beams on different bodies held in its focus were as follows :

A piece

A piece of Roman tile began to melt in 3 seconds, and was ready to drop in 100 seconds.

Chalk fled away in 33 seconds.

A fossil shell calcined in 7 seconds.

Copper ore vitrified in 8 seconds.

Iron ore melted in 24 seconds.

A great tooth of a fish melted in 33 seconds.

Welsh asbestos was a little calcined in 28 seconds.

A king George's halfpenny melted in 16 seconds.

Tin melted in 3 seconds, and had a hole in it in 6.

A bone calcined in 4 seconds, and was vitrified in 33.

A diamond weighing 4 grains lost $\frac{7}{8}$ parts of its weight.

The solar beams are condensed 1700 times in the focus of this mirror (the condensation in the focus being as the area of the mirror is to the area of its focus) and their heat, in the focus, is 433 times as great as the heat of common fire.

The proportional breadth of each colour in the Rain-bow, supposing the whole breadth thereof to be divided into 360 equal parts.

The red, 45 parts; the orange, 27; the yellow, 48; the green, 60; the blue, 60; the indigo, 40; and the violet, 80.

If the flat upper surface of a top be divided into 360 equal parts, all around
its

its edge, and be divided by 7 lines into so many portions or sectors of circles, in the above proportions, and the respective colours be lively painted in these spaces, but so as the edge of each colour may be made nearly like the colour next adjoining, that the separation may not be well distinguished by the eye; and the top be made to spin, all these colours together will appear white. And if a large round black spot be painted in the middle, so as there may be only a broad flat ring of colours around it; the experiment will succeed the better.

Red is the least refrangible of all colours, *orange* the next least, *yellow* the next, *green* the next, *blue* the next, *indigo* the next, and *violet* the most of all.

Mr. Edward Delaval, F. R. S. has found, by experiments on melting different metals with pure glass, that they colour the glass according to their different densities or specific gravities; the most dense giving a red colour to the

R r

glass,

glafs, and the leaft a blue or violet,
Thus, gold melted with glafs makes it
red ; lead melted with glafs, gives it
an orange colour ; filver a yellow ;
copper a green ; and iron a blue.

Colours produced by the mixture of colourless fluids.

Spirit of wine mixed with spirit of vitriol make a *red*.
Solution of mercury mixed with oil of tartar, *orange*.
Solution of sublimate and lime-water, *yellow*.
Tincture of rofes and oil of tartar, *green*.
Solution of copper and spirit of fal-armoniac, *purple*.
Tincture of rofes and spirit of wine, *blue*.
Solution of sublimate and spirit of fal-armoniac, *white*.
Solution of fugar of lead and folution of vitriol, *black*.

Colours produced by the mixture of coloured fluids.

Tincture of faffron, which is yellow,
mixed with tincture of red rofes, make
a *green*.

Tincture of violets, which is blue,
and spirit of fulphur, which is brown,
make a *crimson*.

Tincture

Tincture of red roses, which is red, and spirit of hartshorn which is brownish, make a *blue*.

Tincture of violets, which is blue, and solution of Hungarian vitriol, which is blue, make a *purple*.

Tincture of violets, which is blue, and solution of copper, which is green, make a *violet*.

Tincture of cyanus (blue-bottle flower) which is blue, and spirit of sal armoniac coloured blue, make a *green*.

Solution of Hungarian vitriol, which is blue, and lixivium, which is brown, make a *yellow*.

Solution of Hungarian vitriol, which is blue, and tincture of red roses, make a *black*.

Tincture of cyanus, which is blue, and solution of copper, which is green, make a *red*.

Colours changed, and restored.

Solution of copper, which is *green*, by spirit of nitre is made colourless; and is again restored by oil of tartar.

Limpid infusion of galls is made *black* by a solution of vitriol, and transparent again by oil of vitriol; and then *black* again by oil of tartar.

Tincture of *red* roses is made black by a solution of vitriol, and becomes *red* again by oil of tartar.

A slight tincture of red roses, by spirit of vitriol becomes a fine *red*; then, by spirit of sal armoniac turns green; and then, by oil of vitriol becomes *red* again.

Solution of verdigrease, which is *green* by spirit of vitriol becomes colourless; then, by spirit of sal armoniac becomes *purple*; and then, by oil of vitriol becomes colourless again.

*The quantity of Land and of Water on
the Earth's surface.*

The seas and unknown parts of the Earth (by a measurement of the best maps) contain 160,522,026 square miles; the inhabited parts 38,990,569:
Europe, 4,456,065; Asia, 10,768,823;
7 Africa,

Africa, 9,564,807; and America, 14,110,874. In all, 199,512,595; which is the number of square miles on the whole surface of the Earth.

The weight of the whole Atmosphere.

On a square inch, it is 15 pounds; on a square foot, 2160; on a square yard, 19,440; on a square mile, 60,217,344,000; and on the whole surface of the Earth, and Sea together, 12,014,118,565,447,680,000 pounds.

The surface of the body of a middle sized man is about 14 square feet; and as the weight or pressure of the air is equal to 2160 pounds on every square foot on (or near) the Earth's surface; and as the pressure of the air is equal in all manner of directions, its pressure on the whole body of a middle sized man is equal to 30,240 pounds, or $13\frac{1}{2}$ tons. But, because the spring of the internal air is of equal force with the pressure of the external, the pressure is not felt.

The

The diameter and circumference of the visible part of a cloudy sky.

The greatest distance of the clouds in the horizon at sea is 94 miles from the observer, all around; and consequently, the whole extent or diameter of the horizon, reaching to the clouds, is 188 miles; and the circumference thereof is 590.97 miles.

The velocity of Light.

It has been proved, by the eclipses of Jupiter's Satellites, that light takes 8 minutes of time to come from the Sun to the Earth. And as the Earth's distance from the Sun is 95,000,000 miles in round numbers, 'tis plain that the velocity of light is 11,875,000 miles in a minute, and consequently 197,916 miles in a second; which is 1,486,458 times as swift as the motion of a cannon ball, and 10,440 times as swift as the Earth moves in its annual orbit.

The

The velocity of sound.

According to Dr. *Halley*, Mr. *Flamsteed*, and Mr. *Derham*, sound moves 1142 feet in a second of time, 68,520 feet in a minute, and 4,111,200 miles in an hour.

Hence we may know how far a thunder cloud is from us, if we have a watch that shews seconds. Thus, suppose there were four seconds from the moment we see the flash of *lightning* to the moment we hear the clap of *thunder*, 'tis plain that the cloud which produced the thunder is four times 1142 feet, or 4568 feet from us; which is about four-fifths of a mile.

The cause of the ebbing and flowing of the sea at the same time on opposite sides of the globe.

The reason why the tides rise on the side of the Earth which is at any time turned towards the Moon, is plain to every one; because her attraction must
occasion

occasion a swelling of the waters toward her on that side: but the cause of as great a swell, at the same time, on the opposite side of the Earth, which is then turned away from the Moon, has been very hard to account for; because the rising of the tide there is in a direction quite contrary to the attraction of the Moon. But this difficulty is immediately removed, when we consider, that all bodies moving in circles have a centrifugal force, or constant tendency to fly off from the centers of the circles they describe; and this centrifugal force is always in proportion to the distance of the body from the center of its orbit, and the velocity with which it moves therein.

When the body is large, the side of it which is farthest from the center of its orbit will have a greater degree of centrifugal force than the center of the body has; and the side of it which is nearest the center of its orbit will have a less degree of centrifugal force than its center has.

As

As the Moon goes round the Earth every month in her orbit, the Earth also goes round an orbit every month, which is as much less than the Moon's orbit, as the quantity of matter in the Moon is less than the quantity of matter in the Earth, which is 40 times. For, by the laws of nature, when a small body moves round a great **one**, in free and open space, both **these** bodies must move round the common center of gravity between them.

The Moon's mean distance from the Earth's center is 240,000 English miles: divide therefore this distance by 40, the difference between the quantity of matter in the Earth and Moon; and the quotient will be 6000 miles, which is the distance of the common center of gravity (between the Earth and Moon) from the center of the Earth.

Now, as the Earth and Moon move round the common center of gravity between them, once every month; 'tis plain, that whilst the Moon moves round her orbit, at 240,000 miles from the

S f Earth's

Earth's center, the center of the Earth describes a circle of 6000 miles radius, round the center of gravity between the Earth and the Moon; the Moon's attraction balancing the centrifugal force of the Earth at its center.

The diameter of the Earth is 8000 miles, in round numbers, and consequently its semidiameter is 4000 : so that the side of the Earth, which is at any time turned toward the Moon, is 4000 miles nearer the common center of gravity between the Earth and Moon than the Earth's center is ; and the side of the Earth, which is then farthest from the Moon, is 4000 miles farther from the center of gravity between the Earth and Moon than the Earth's center is at that time.

Therefore, the radius of the circle described by the parts of the Earth which come about toward the Moon, by the Earth's diurnal motion, is 2000 miles; the radius of the circle described by the Earth's center is 6000 ; and the radius of the circle described by those
parts

parts of the Earth which, in revolving on its axis, are furthest from the Moon, is 10,000 miles.

The centrifugal forces of the different parts of the Earth being directly as their distances from the abovementioned common center of gravity, round which both the Earth and Moon move, these forces may be expressed by 2000 for the side of the Earth nearest the Moon, by 6000 for the Earth's center, and by 10,000 for the side of the Earth which is farthest from the Moon.

But the Moon's attraction is greatest on the side of the Earth next her, where the centrifugal force or tendency to fly off from the common center of gravity (and consequently, from the Moon) is least; and therefore, the tides must rise on the side of the Earth which is nearest the Moon, by the excess of the Moon's attraction.

As her attraction balances the centrifugal force at the Earth's center, 'tis plain that the centrifugal force of the side of the Earth which is farthest from

the Moon is greater than her attraction; and therefore, the tides will rise as high upon that side from the Moon, by the excess of the centrifugal force, as they rise on the side next her by the excess of her attraction. And as the Earth is in constant motion on its axis, so as that any given meridian revolves from the Moon to the Moon again in 24 hours, $50\frac{1}{2}$ minutes, each place will come to the two eminences of water, under and opposite to the Moon, in 24 hours, $50\frac{1}{2}$ minutes, or have two tides of flood and two of ebb in that time. For, as much as the waters rise above the common level of the surface of the sea, under and opposite to the Moon, so much they must fall below that level half way between the highest places; or at 90 degrees from them.

On these principles, it is equally easy to account for the rising of the tides, at the same time, on both sides of the Earth: and this rising is made evident to sight in my Lecture on the central forces; and the principles on which it depends

depends are made obvious to the understandings of all the observers.

*Surprising properties of numbers, placed
in squares and circles.*

I have seen several different kinds of (what is generally called) magic squares; but have lately got a magic square of squares and a magic circle of circles of a very extraordinary kind, from Dr. BENJAMIN FRANKLIN of *Philadelphia*, with his leave to publish them. The magic square goes far beyond any thing of the kind I ever saw before; and the magic circle (which is the first of the kind I ever heard of, or perhaps any one besides) is still more surprising. What the Doctor's rules are, for disposing of the different numbers so, as that they shall have the following properties, I know nothing of: and perhaps the reason may be, that I have not ventured to ask him; although I never saw a more communicative man in my life. The plates of these are at the end of the book.

PLATE I.

A magic square of squares.

The great square is divided into 256 small squares, in which all the numbers from 1 to 256 are placed, in 16 columns, which may be taken either horizontally or vertically. The properties are as follows:

1. The sum of the sixteen numbers in each column, vertical and horizontal, is 2056.

2. Every half column, vertical and horizontal, makes 1028, or half of 2056.

3. Half a diagonal ascending, added to half a diagonal descending, makes 2056; taking these half diagonals from the ends of any side of the square to the middle thereof; and so reckoning them either upward, or downward; or sidewise from left to right hand, or from right to left.

4. The same with all the parallels to the half diagonals, as many as can be

drawn in the great square: for any two of them being directed upward and downward, from where they begin to where they end, their sums will make 2056. The same downward and upward from where they begin to where they end; or all the same if taken sideways to the middle, and back to the same side again.

N. B. One set of these half diagonals and their parallels, is drawn in the square upward and downward. Another such set may be drawn from any of the other three sides.

5. The four corner numbers in the great square added to the four central numbers therein, make 1028; equal to the half sum of any vertical or horizontal column, which contains 16 numbers; and equal to half a diagonal or its parallel.

6. If a square hole (equal in breadth to four of the little squares) be cut in a paper, through which any of the sixteen little squares in the great square may be seen, and the paper be laid on the
great

great square ; the sum of all the 16 numbers, seen through the hole, is equal to the sum of the sixteen numbers in any horizontal or vertical column, *viz.* to 2056.

PLATE II.

A magic circle of circles.

This circle is composed of a series of numbers, from 12 to 75 inclusive, divided into eight concentric circular spaces, and ranged in eight radii of numbers, with the number 12 in the center ; which number, like the center, is common to all these circular spaces, and to all the radii.

The numbers are so placed, that the sum of all those in either of the concentric circular spaces above-mentioned, together with the central number 12, make 360 ; equal to the number of degrees in a circle.

The numbers in each radius also, together with the central number 12, make just 360.

The

The numbers in half of any of the above circular spaces, taken either above or below the double horizontal line, with half the central number 12, make 180; equal to the number of degrees in a semicircle.

If any four adjoining numbers be taken, as if in a square, in the radial divisions of these circular spaces; the sum of these, with half the central number, make 180.

There are, moreover, included four sets of other circular spaces, bounded by circles which are excentric with respect to the common center; each of these sets containing five spaces. The centers of the circles which bound them are at *A*, *B*, *C*, and *D*. The set whose center is at *A* is bounded by dotted lines; the set whose center is at *C* is bounded by lines of short unconnected strokes; and the set round *D* is bounded by lines of unconnected longer strokes, to distinguish them from one another. In drawing this figure by hand, the set of concentric circles should be drawn

T t

with

with black ink ; and the four different sets of excentric circles with four kinds of ink of different colours; as blue, red, yellow, and green, for distinguishing them readily from one another.

These sets of excentric circular spaces intersect those of the concentric, and each other: and yet, the numbers contained in each of the excentric spaces, taken all around through any of the 20, which are excentric, make the same sum as those in the concentric; namely 360, when the central number 12 is added. Their halves also, taken above or below the double horizontal line, with half the central number, make 180.

Observe, that there is not one of the numbers but what belongs at least to two of the circular spaces; some to three, some to four, some to five: and yet they are all so placed, as never to break the required number 360, in any of the 28 circular spaces within the primitive circle.

To bring these matters in view, I have taken out all the numbers as
above-

above-mentioned : and have placed them in separate columns, as they stand around both the concentric and excen- tric circular spaces, always beginning with the outermost, and ending with the innermost of each set ; and also the numbers as they stand in the eight radii, from the circumference to the center : the common central number 12 being placed the lowest in each column.

1. In the eight concentric circular spaces.

14	72	23	65	21	67	12	74
25	63	16	70	18	68	27	61
30	56	39	49	37	51	23	58
41	47	32	54	34	52	43	45
46	40	55	33	53	35	44	42
57	31	48	38	50	36	59	29
62	24	71	17	69	19	60	26
73	15	64	22	66	20	75	13
12	12	12	12	12	12	12	12
360	360	360	360	360	360	360	360

2. In the eight radii.

14	25	30	41	46	57	62	73
72	63	56	47	40	31	24	15
23	16	39	32	55	48	71	64
65	70	49	54	33	38	17	22
31	18	37	34	53	50	69	66
67	68	51	52	35	36	19	20
12	27	28	43	44	59	60	75
74	61	58	45	42	29	26	13
12	12	12	12	12	12	12	12
360	360	360	360	360	360	360	360

3. In the five excentric circular spaces whose center is at A.	14	32	23	65	21
	03	16	70	18	68
	39	49	37	51	28
	54	34	52	43	45
	33	53	35	44	42
	48	38	50	36	59
	24	71	17	69	19
	73	15	64	22	66
	12	12	12	12	12
	360	360	360	360	360
4. In the five excentric circular spaces whose center is at B.	30	56	39	49	37
	47	32	54	34	52
	55	33	53	35	44
	38	50	36	59	29
	17	69	19	60	26
	64	22	66	20	75
	72	23	65	21	67
	25	63	16	70	18
	12	12	12	12	12
	360	360	360	360	360
5. In the five excentric circular spaces whose center is at C.	46	40	55	33	53
	31	48	38	50	36
	71	17	69	19	60
	22	66	20	75	13
	65	21	67	12	74
	16	70	18	68	27
	56	39	49	37	51
	41	47	32	54	34
	12	12	12	12	12
	360	360	360	360	360
6. In the five excentric circular spaces whose center is at D.	62	24	71	17	59
	15	64	22	66	20
	23	65	21	67	12
	70	18	68	27	61
	49	37	51	28	58
	32	54	34	52	43
	40	55	33	53	35
	57	31	48	38	50
	12	12	12	12	12
	360	360	360	360	360

If now, we take any four numbers, as in a square form, either from N° 1. N° 2. (as suppose from N° 1.) as in the margin; and add half the central number 12 to them, the sum will be 180; equal to half the numbers in any circular space, taken above or below the double horizontal line: and equal to the number of degrees in a semicircle. Thus, 14, 72, 25, 63, and 6, make 180.

14	72
25	63
	6

A List

A List of the Apparatus on which Mr. Ferguson reads his Course of twelve Lectures on Mechanics, Hydrostatics, Hydraulics, Pneumatics, Dialing and Astronomy.

The numbers relate to the Lectures read on the machinery to which they are prefixed.

I.

Simple machines for demonstrating the powers of the lever, the wheel and axle, the pullies, the inclined plane, the wedge, and the screw.

A compound engine in which all these simple machines work together.

A working model of the great crane at *Bristol*, which is reckoned to be the best crane in Europe.

A working model of a crane that has four different powers, to be adapted to the different weights intended to be raised : invented by Mr. *Ferguson*.

A pyrometer that makes the expansion of metals by heat visible to the 90 thousandth part of an inch ; so as to be seen by the bare eye at two feet distance from the machine.

II.

Simple machines for shewing the center of gravity of bodies, and how far a tower may incline without danger of falling.

A double cone that seemingly rolls up-hill of itself, whilst it is actually descending.

A machine made in the figure of a human creature, that tumbles backward, by continually oversetting the center of gravity.

Models of wheel-carriages ; some with broad wheels, others with narrow ; some with large wheels, and others with small : for proving experimentally which sort is the best.

A machine for shewing what degree of power is sufficient to draw a loaded
cart

cart or waggon up-hill; when the quantity of weight to be drawn up, and the angle of the hill's height, are known.

A machine for diminishing friction; and shewing that the friction is not in proportion to the quantity of the surface that either rubs or rolls; but in proportion to the weight with which the machine is loaded.

A model of a most curious silk-reel, invented by Mr. *Verrier* near *Wrington* in *Somersetshire*.

A large working model of a water-mill for sawing timber.

A model of a hand-mill for grinding corn.

A model of a water-mill, for winnowing and grinding corn, drawing up the sacks, and boulding the flour.

A machine for demonstrating that the power of the wind, on wind-mill sails, is as the square of the velocity of the wind.

A model

A model of the engine by which the piles were driven for a foundation to the piers of *Westminster* bridge.

III.

A machine for shewing that fluids weigh as much in their own elements as they do in air.

A machine for shewing that, on equal bottoms, the pressure of fluids is in proportion to their perpendicular heights; let their quantities be ever so great or ever so small.

Machines for shewing that fluids press equally in all manner of directions.

A machine for shewing how an ounce of water in a tube may be made to raise and support sixteen pounds weight of lead.

A machine for shewing, that, at equal heights, the smallest quantity of water whatever will balance the greatest quantity whatever, if the columns join at bottom.

U u

A ma-

A machine for shewing how solid lead may be made to swim in water, and the lightest wood to sink in water.

Machines for shewing and demonstrating the hydrostatical paradox.

A machine for demonstrating that the weight of the quantity of water displaced by a ship is equal to the whole weight of the ship and cargo.

Machines for shewing the working of syphons, and the *Tantalus's* cup.

A large machine for shewing the cause and explaining the phenomena of ebbing and flowing wells, and of intermitting and reciprocating springs.

IV.

Machines for shewing that when solid bodies are immersed and suspended in fluids, the solid loses as much of its weight as its bulk of the fluid weighs ; and that the weight lost by the solid is imparted to the fluid.

A hydrostatic balance, for shewing the specific gravities of bodies, and detecting counterfeit gold or silver.

A working model of *Archimedes's* spiral pump.

Glass models for shewing the structure and operations of sucking, forcing and lifting pumps.

A working model of a quadruple pump-mill for raising water by means of water turning a wheel.

A working model of the *Persian* wheel for raising water.

A model of the great hydraulic engine under *London* bridge, that goes by the tides, and raises water by forcing pumps.

A working model of Mr. *Blakey's* intended engine for raising water by means of fire.

V. and VI.

An air-pump, with a great apparatus to it, for experiments shewing the weight and spring of the air.

VII.

An electrical machine, with a very large apparatus, for shewing a great

variety of curious and entertaining experiments, many of which are entirely new.

VIII.

A whirling table, for explaining and demonstrating the laws by which the planets move, and are retained in their orbits: that the Sun and all the planets move round their common center of gravity: that the Earth and Moon move round their common center of gravity once every month: that the Earth moves round the Sun, in common with the rest of the planets, and turns round its own axis: that the power of gravity diminishes in proportion as the square of the distance from the attracting body increases: that a double velocity in any orbit would require a quadruple power of gravity to retain the body in that orbit: that the squares of the periodical times in which the planets move round the Sun are in proportion to the cubes of their distances from the Sun. A plain experimental demonstration of
the

the doctrine of the tides ; and the cause of their rising equally high, at the same time, on opposite sides of the Earth.

IX, X, XI, and XII.

A machine for shewing the motions of the comets.

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Pl. II.

A Magic Square of Squares.

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

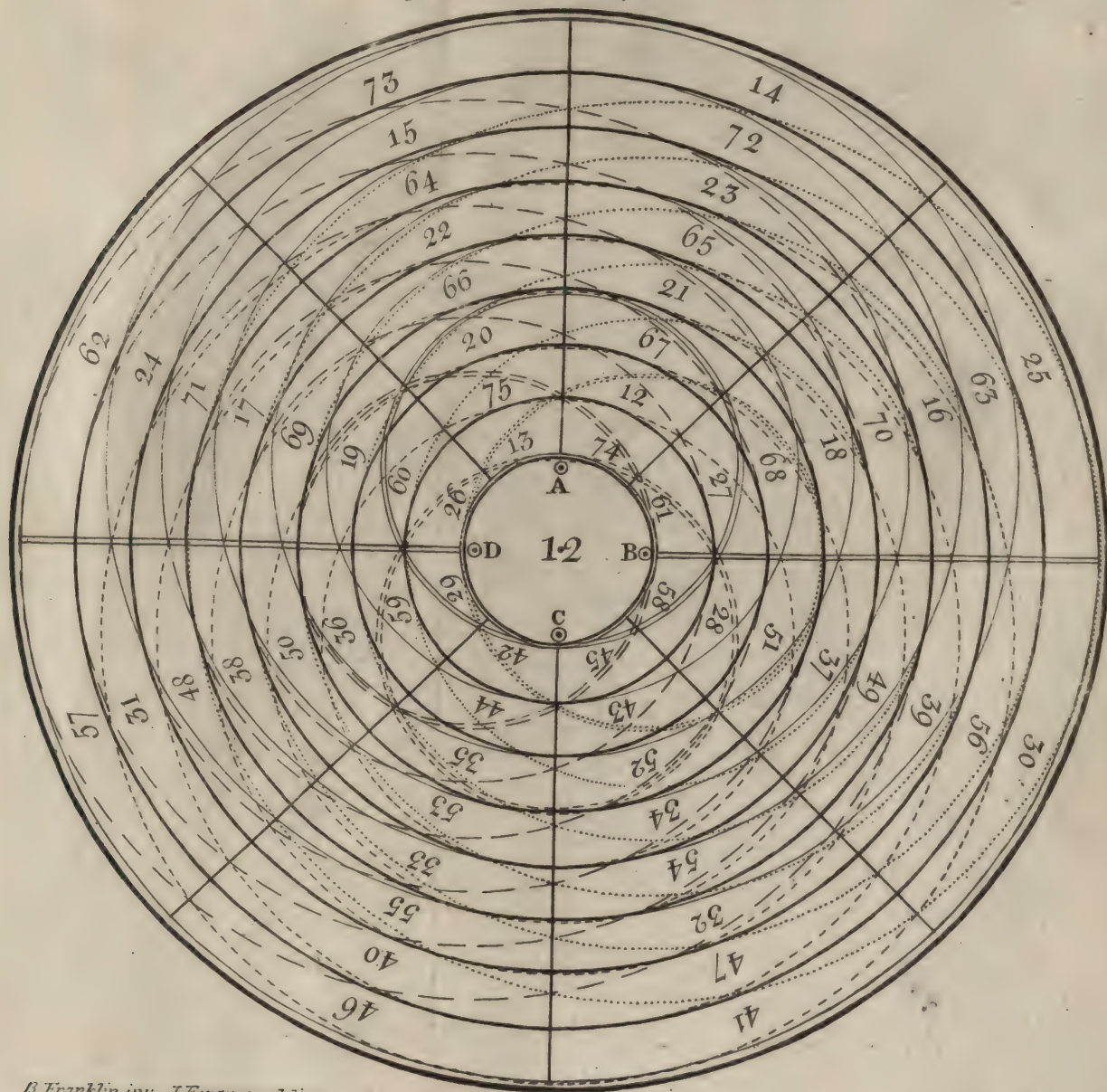
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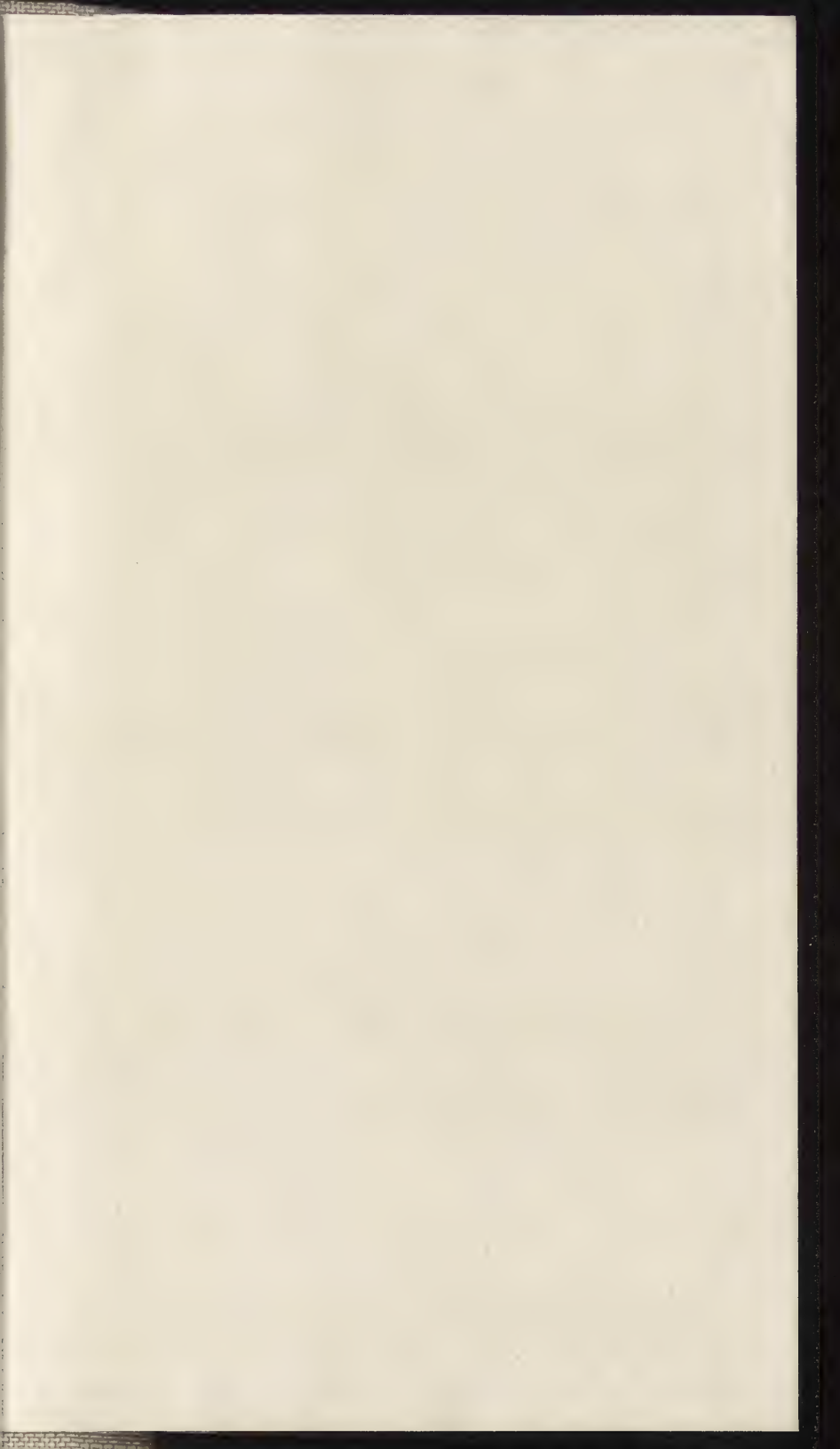
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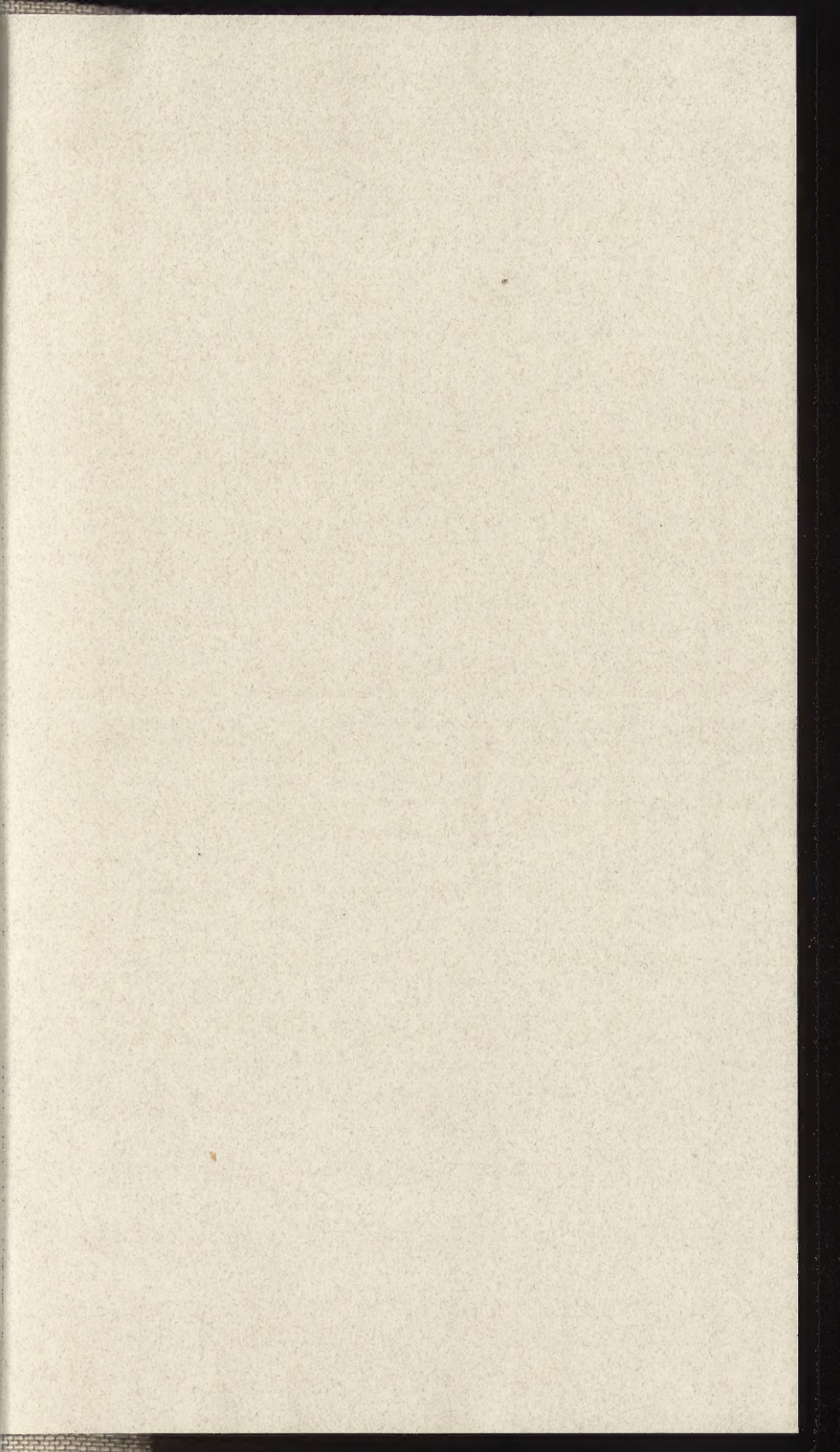
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